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THE ANALYSIS OF SOUND PROPAGATION IN JET ENGINE DUCTS USING THE FINITE DIFFERENCE METHOD

ANALYSIS AND OPTIMIZATION BRANCH
STRUCTURES & DYNAMICS DIVISION

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In this report, the author derives the partial differential equations which describe sound propagation in jet engine ducts and then presents a finite difference approach for solving these equations. Also included is a computer program listing, sample input and sample output. The program can handle uniform rectangular and cylindrical ducts with or without uniform flow. In addition, if a mapping function which maps a nonuniform duct to a uniform duct is specified, the program can determine sound fields in nonuniform ducts in the absence of flow.

FOREWORD

This report describes work performed in the Air Force Flight Dynamics Laboratory under Project 2304N112, Theoretical Duct Acoustics. This is an interim report on work carried out between November 1975 and November 1978.

The author thanks Ms. Jean Schwab for her preliminary editing, proofreading and setting up and running sample cases. The author thanks also Ms. Lynn Curtis for her typing in the preparation of this report.

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INTRODUCTION

In this report we consider the problem of computing sound propagation in ducts. These ducts are models of the interior of jet engines. Since the largest source of jet engine noise is fan noise, there has been an increase in interest in duct acoustics in recent years. We will consider ducts of uniform cross section with or without mean flow and ducts with variable cross section not containing flow. The ducts are either two dimensional or axially symmetric three dimensional. To compute the sound field within one of these ducts, the momentum and continuity equations are linearized and combined to yield a linear, second order partial differential equation in the acoustic pressure. The boundary conditions consist of a specified pressure distribution at the duct entrance, a vanishing normal derivative at the centerline, and a specified acoustic impedance at the exit and at the outer wall of the duct which corresponds to an acoustic lining. These equations are then solved using the finite difference method. The large system of algebraic equations which result from using finite differences is solved using an out-of-core block tri-diagonal system solver on the CDC Cyber 74 computer. The acoustic velocities are then determined by integrating the linearized momentum equations and then used to compute the acoustic power at the entrance and exit of the duct. These are then used to determine the attenuation resulting from a particular wall lining.

For a nonuniform duct the additional step of mapping the nonuniform geometry to a uniform duct is required before the finite difference method is applied to obtain the solution. We use a conformal map so

that right angles in the original coordinate system become right angles in the mapped coordinate system. The mapped equations are then solved using finite differences as indicated.

In Section II, we will describe in more detail the equations and boundary conditions which need to be solved and give the expressions used to compute the acoustic power. In Section III, we discuss the conformal mapping procedure and derive the mapped equations and boundary conditions. In Section IV, we derive the finite difference equations for both the original and mapped equations, and indicate the method used to solve the resultant system of algebraic equations. In Section V, we describe the computer program DUCT including minor program modifications for extra capability, and in Section VI we give samples with input and output.

SECTION II

ANALYTIC MODEL

Differential Equations

If viscous and heat transfer terms are neglected, the equations of motion for an ideal gas are

Momentum $$\rho' \, D(V')/Dt = -\nabla p' \qquad (1a)$$

Continuity $$D(\rho')/Dt + \rho'(\nabla \cdot V') = 0 \qquad (1b)$$

Energy and state $$p' = \text{const } \rho'^{\alpha} \qquad (1c)$$

where ρ' is the density, V' is the velocity vector, and p' is the pressure. If it is assumed that the pressure perturbations are small compared to the average pressure and that the steady velocities are negligible, Eq. (1) becomes

$$\partial\hat{p}/\partial t = -\rho_o c^2 \nabla\cdot\hat{V} \quad (2a)$$

$$\rho_o \; \partial\hat{V}/\partial t = -\nabla\hat{p} \quad (2b)$$

When combined, Eqs. (2) reduce to the wave equation

$$\nabla^2\hat{p} = (1/c^2)\; \partial^2\hat{p}/\partial t^2 \quad (3)$$

and, with time dependence of the form $e^{i\omega t}$ [that is, $\hat{p}(X,Y,Z,t) = e^{i\omega t}p(X,Y,Z)$], Eq. (3) becomes the Helmholtz equation for the pressure p

$$\nabla^2 p + (\omega^2/c^2)p = 0 \quad (4)$$

Analyzing only one frequency for fan noise is reasonable since noise at different frequencies can be superimposed. In nondimensional coordinates,[1,2,3,4] Eq. (4) becomes

$$\nabla^2 p + (2\pi\eta)^2 p = 0 \quad (5)$$

with $\eta = R\omega/(2\pi c)$ for a cylindrical duct (R being the duct radius), $\eta = H\omega/(2\pi c)$ for a rectangular duct (H being the duct height). In two dimensions, for rectangular ducts, Eq. (5) is

$$\partial^2 p/\partial x^2 + \partial^2 p/\partial y^2 + (2\pi\eta)^2 p = 0 \quad (6)$$

whereas, in three dimensions with axial symmetry, Eq. (5) becomes

$$\partial^2 p/\partial x^2 + \partial^2 p/\partial r^2 + (1/r)\; \partial p/\partial r + (2\pi\eta)^2 p = 0 \quad (7)$$

where x is the axial direction and r is the radial direction. In the mean flow case, the steady velocity in the axial direction is assumed to be a constant U (and no longer negligible), yielding the dimensionless axially symmetric reduced wave equation[3]

$$(1-M^2)\,\partial^2 p/\partial x^2 + \partial^2 p/\partial r^2 + (1/r)\,\partial p/\partial r - 4\pi\eta M i\,\partial p/\partial x + (2\pi\eta)^2 p = 0 \tag{8}$$

where $M = U/c$ is the Mach number and $\eta = R\omega/(2\pi c)$ is the dimensionless frequency, R being the duct radius.

Boundary Conditions

To absorb acoustic energy within the duct, the outer walls are lined with Helmholtz resonators. The Helmholtz resonator can consist of a perforated sheet placed at distance d from a solid backing sheet. The acoustic impedance of this wall lining is determined by the hole diameters and backing depth.

If Z is the acoustic impedance at the outer wall, then

$$Z = p/n\cdot V$$

where n is the exterior normal to the outer wall. For no flow, $i\omega\rho_o V = -\nabla p$, so

$$iZ/\omega\rho_o = p/n\cdot\nabla p$$

or

$$\partial p/\partial n + i\omega\rho_o\, p/Z = 0 \tag{9}$$

at the wall. [Note that $\partial p/\partial n = n\cdot\nabla p$ by definition.] For the case of uniform flow the continuity of particle displacement is used rather than the continuity of particle velocity to derive the dimensionless boundary condition

$$\partial p/\partial n = -2\pi\eta i p/Z_w - (2M/Z_w)\,\partial p/\partial x + iM^2/(2\pi\eta Z_w)\,\partial^2 p/\partial x^2 \tag{10}$$

at the outer wall (see Ref. 5 for the derivation). For a variable wall lining (such as a three-sectional lining) Z_w is a function of x instead of being constant, as is the case for a uniform lining. The remaining boundary conditions for a cylindrical duct are

$$p(0,r) = f(r) \qquad \text{at the entrance} \tag{11a}$$

$$\partial p/\partial n = 0 \qquad \text{at } r = 0 \text{ (the centerline)} \tag{11b}$$

$$\partial p/\partial n + 2\pi\eta i p/Z_e = 0 \qquad \text{at the exit} \tag{11c}$$

For the rectangular duct the only change in the boundary condition is that $p(0,y) = f(y)$ is specified as an entrance condition.

<u>Sound-Power Attenuation</u>

For both the no-flow and uniform flow cases, the expression used to compute the dimensionless axial intensity[3,4] is

$$I(x,r) = \text{real } (p^*u)/(2\pi\eta) + M[p^*p + (u^*u + v^*v)/(2\pi\eta)^2]/2 \tag{12}$$

where u is the velocity in the axial direction, v is the velocity in the radial direction, and the asterisk denotes complex conjugation. For the no flow case, of course, $M = 0$ and

$$I = \text{real } (p^*u)/(2\pi\eta) \tag{13}$$

To compute u and v, once p is known, the dimensionless x-momentum and r-momentum equations are used:

$$u = i\ \partial p/\partial x + i\ M/(2\pi\eta)\ \partial u/\partial x \tag{14a}$$

$$v = i\ \partial p/\partial y + i\ M/(2\pi\eta)\ \partial v/\partial x \tag{14b}$$

The initial condition used to solve for u is that $u = 2\pi\eta p$ at the exit. Then the irrotationality condition $\partial u/\partial y = \partial v/\partial x$ and Eqs. (14) are used to solve for v. Once the intensity is known the dimensionless acoustic power is computed as

$$E_x = \int_A I(x,r)dA = \int_0^1 I(x,r)rdr \tag{15}$$

for a cylindrical duct, or

$$E_x = \int_0^1 I(x,y)dy \tag{16}$$

for a rectangular duct. The sound attenuation is then determined as

$$dB = 10 \log_{10} [E_x/E_o] \tag{17}$$

where E_o is the sound power at the entrance and E_x is the sound power at the axial point x.

SECTION III

CONFORMAL MAPPING

The well-known Riemann Mapping Theorem establishes that an arbitrary simply connected domain in the plane can be mapped conformally onto an open rectangle with vertices (0,1), (0,-1), (X,-1), and (X,1). (For a diagram of such a map, see Fig. 1. Other examples of variables of ducts are given in Figure 2.) If the map is of the form

$$z = z(x,y);\ r = r(x,y)$$

then, for it to be conformal, the Cauchy-Riemann equations

$$\partial z/\partial x = \partial r/\partial y;\ \partial z/\partial y = -\ \partial r/\partial x$$

must be satisfied.

Assuming axial symmetry, Eq. (4) may be rewritten as

$$\partial^2 p/\partial z^2 + \partial^2 p/\partial r^2 + (1/r)\ \partial p/\partial r + (\omega/c)^2 p = 0$$

where z is the axial coordinate and r is the radial coordinate. In transformed coordinates this equation becomes

$$0 = \frac{\partial^2 \hat{p}}{\partial x^2} + \frac{\partial^2 \hat{p}}{\partial y^2} - \frac{1}{r(x,y)} \frac{\partial z(x,y)}{\partial y} \frac{\partial \hat{p}}{\partial x} + \frac{1}{r(x,y)} \frac{\partial z(x,y)}{\partial x} \frac{\partial \hat{p}}{\partial y} + (2\pi\eta)^2 \left\{ \left(\frac{\partial z(x,y)}{\partial x} \right)^2 + \left(\frac{\partial z(x,y)}{\partial y} \right)^2 \right\} \hat{p} \tag{18}$$

where $\hat{p}(x,y) = p(z,r)$ and $\eta = R\omega/(2\pi c)$ is the dimensionless frequency. The boundary conditions (9) and (11) become

$$\hat{p}(0,y) = f(z(0,y),r(0,y)) = g(y) \quad \text{at the entrance} \tag{19a}$$

$$\frac{\partial \hat{p}}{\partial y} = 0 \quad \text{at } y = 0 \tag{19b}$$

$$\frac{\partial \hat{p}}{\partial y} = -2\pi\eta i p \left[\left(\frac{\partial z}{\partial x} \right)^2 + \left(\frac{\partial z}{\partial y} \right)^2 \right]^{1/2} \Big/ Z_y \quad \text{at } y = 1 \tag{19c}$$

$$\frac{\partial \hat{p}}{\partial x} = -2\pi\eta i p \left[\left(\frac{\partial z}{\partial x} \right)^2 + \left(\frac{\partial z}{\partial y} \right)^2 \right]^{1/2} \Big/ Z_x \quad \text{at } x = X. \tag{19d}$$

Finite difference solutions will be obtained in Sec. IV. (See Ref. 6 for a more complete description.)

Example

For positive real $\alpha < \frac{\pi}{2}$, the cone enclosed in the (z,r) plane by the lines $r = (\sin\alpha)z/\cos\alpha$ and $r = -(\sin\alpha)z/\cos\alpha$ and by arcs of the circles $z^2 + r^2 = R^2$ and $z^2 + r^2 = R^2 e^{2\alpha X}$ is the region considered.

The transformation defined by

$$z = Re^{\alpha x} \cos \alpha y; \; r = Re^{\alpha x} \sin \alpha y \tag{20}$$

maps the open rectangle onto the prescribed cone. With $z(x,y)$ and $r(x,y)$ defined by Eq. (20), the finite difference equations corresponding to Eqs. (18) and (19) were solved on a rectangle using the method described above.

SECTION IV

FINITE DIFFERENCE SOLUTIONS OF ANALYTIC MODEL

To obtain solutions of Eqs. (6-8) in the case of a rectangular duct, a no-flow cylindrical duct, or a cylindrical duct with uniform flow, respectively, the method of finite differences was used. The boundary conditions (10) and (11) must be included for the problems to be well posed. The finite difference equations may be summarized as follows (see Refs. 2, 3, 6, and especially 7). Set

$$\delta x = X/(n+1)$$

$$\delta r = 1/(m+1)$$

$$x_j = j\delta x \qquad j = 1, \cdots, n$$

$$r_k = k\delta r \qquad k = 1, \cdots, m$$

and

$$p_{j,k} = p(x_j, y_k)$$

where m and n are positive integers. Then for a cylindrical duct the difference equation at a point not adjacent to the boundary is

$$(1-M^2)(p_{j+1,k} - 2p_{j,k} + p_{j-1,k})/(\delta x)^2$$

$$+ (p_{j,k+1} - 2p_{j,k} + p_{j,k-1})/(\delta r)^2 + (p_{j,k+1} - p_{j,k-1})/(2(\delta r)r_k)$$

$$- 4\pi\eta Mi(p_{j+1,k} - p_{j-1,k})/(2\delta x) + (2\pi\eta)^2 p_{j,k} = 0.$$

Collecting terms, the equation becomes

$$[2 + 2(1-M^2)(\delta r)^2/(\delta x)^2 - (2\pi\eta\delta r)^2]\ p_{j,k} - p_{j,k+1}[1 + 1/(2k)]$$

$$- p_{j,k-1}[1 - 1/(2k)] - p_{j+1,k}[1-M^2 - 2\pi\eta Mi\delta r^2/\delta x]$$

$$- p_{j-1,m}[1-M^2 + 2\pi\eta Mi\delta r^2/\delta x] = 0 \text{ for } 1 \le j \le n-1,\ 2 \le k \le m-1.$$

Similar expressions are obtained for $j = n$, $k = 1$, $k = m$, which correspond to the boundary conditions (10) and (11).

To obtain a solution of Eqs. (18) and (19) on a cylinder, the following finite difference scheme is employed.

Set

$$\delta x = X/(n + 1),$$

$$\delta y = 1/(m + 1),$$

$$x_j = j\ \delta x, \qquad j = 1, \cdots, n,$$

$$y_k = k\ \delta y, \qquad k = 1, \cdots, m,$$

$$z_{j,k} = \frac{\partial z}{\partial y}(x_j, y_k)$$

$$\hat{z}_{j,k} = \frac{\partial z}{\partial x}(x_j, y_k), \qquad j = 1, \cdots, n,$$

$$r_{j,k} = r(x_j, y_k), \qquad k = 1, \cdots, m,$$

and

$$p_{j,k} = p(x_j, y_k),$$

where m and n are positive integers. Then the difference equation is

$$
\begin{aligned}
&(p_{j+1,k} - 2p_{j,k} + p_{j-1,k})/(\delta x)^2 \\
&+ (p_{j,k+1} - 2p_{j,k} + p_{j,k-1})/(\delta y)^2 \\
&- \frac{z_{j,k}}{r_{j,k}} (p_{j+1,k} - p_{j-1,k})/(2\delta x) \\
&+ \frac{\hat{z}_{j,k}}{r_{j,k}} (p_{j,k+1} - p_{j,k-1})/(2\delta y) \\
&+ (2\pi\eta)^2 (\hat{z}_{j,k}{}^2 + z_{j,k}{}^2) p_{i,j} = 0.
\end{aligned}
$$

Collecting terms, the equation becomes

$$
\begin{aligned}
&\left[2 + 2(\delta y)^2/(\delta x)^2 - (2\pi\eta\delta y)^2(\hat{z}_{j,k}{}^2 + z_{j,k}{}^2)\right] p_{j,k} \\
&- p_{j,k+1}\left(1 + \frac{\hat{z}_{j,k}}{r_{j,k}}\,\delta y/2\right) - p_{j,k-1}\left(1 - \frac{\hat{z}_{j,k}}{r_{j,k}}\,\delta y/2\right) \\
&- p_{j+1,k}\left[(\delta y)^2/(\delta x)^2 - \frac{z_{j,k}}{r_{j,k}}\frac{(\delta y)^2}{2\delta x}\right] \\
&- p_{j-1,k}\left[(\delta y)^2/(\delta x)^2 + \frac{z_{j,k}}{r_{j,k}}\frac{(\delta y)^2}{2\delta x}\right] \\
&= 0 \text{ for } 1 \le j \le n - 1,\ 2 \le k \le m - 1.
\end{aligned}
$$

Similar expressions are obtained for $j = n$, $k = 1$, $k = m$, which correspond to the boundary conditions (19).

For a reasonable number of mesh points, say 50 in the x direction and 20 in the y direction, the linear system of equations to be solved consists of 1000 unknown $p_{j,k}$'s to be determined. Moreover, the $p_{j,k}$'s are complex numbers because of the boundary

conditions, so there are 2000 unknown real numbers to compute. Standard iterative methods will not work for this problem, for the system is neither positive definite nor symmetric. Therefore, a direct method was used to solve the system, taking advantage of its sparseness.

The set of difference equations forms a system of linear equations which can be written in matrix form as $MP = F$ where M is the finite difference matrix, P is the unknown pressure matrix and F is the matrix containing the prescribed pressures.

M has the following form:

$$\begin{matrix}
0 & \dots & 0 & -a & 0 & 0 & 0 & 0 & \dots & 0 & -1 & b & -1 & 0 & 0 & 0 & 0 & \dots & 0 & -c & 0 & 0 & 0 & 0 & \dots & 0 \\
0 & \dots & 0 & 0 & -a & 0 & 0 & 0 & \dots & 0 & 0 & -1 & b & -1 & 0 & 0 & 0 & \dots & 0 & 0 & -c & 0 & 0 & 0 & \dots & 0 \\
0 & \dots & 0 & 0 & 0 & -a & 0 & 0 & \dots & 0 & 0 & 0 & -1 & b & -1 & 0 & 0 & \dots & 0 & 0 & 0 & -c & 0 & 0 & \dots & 0 \\
0 & \dots & 0 & 0 & 0 & 0 & -a & 0 & \dots & 0 & 0 & 0 & 0 & -1 & b & -1 & 0 & \dots & 0 & 0 & 0 & 0 & -c & 0 & \dots & 0
\end{matrix}$$

which can be written as a block-tridiagonal matrix

$$\begin{matrix}
B_1 & C_1 & 0 & 0 & \dots & 0 \\
A_2 & B_2 & C_2 & 0 & \dots & 0 \\
0 & A_3 & B_3 & C_3 & & 0 \\
0 & \dots & & & A_n & B_n
\end{matrix}$$

where A, B, C are square matrices which fit into the pattern.

M can now be factored as

$$M = L * U$$

or

$$\begin{pmatrix}
B_1 & C_1 & 0 & 0 & \dots & 0 \\
A_2 & B_2 & C_2 & 0 & \dots & 0 \\
0 & A_3 & B_3 & C_3 & & 0 \\
0 & \dots & & & A_n & B_n
\end{pmatrix} = \begin{pmatrix}
I & 0 & 0 & \dots & 0 \\
L_2 & I & 0 & \dots & 0 \\
0 & L_3 & I & \dots & 0 \\
0 & 0 & \dots & L_n & I
\end{pmatrix} \begin{pmatrix}
U_1 & C_1 & 0 & 0 & \dots & 0 \\
0 & U_2 & C_2 & 0 & \dots & 0 \\
0 & 0 & U_3 & C_3 & \dots & 0 \\
0 & 0 & & \dots & 0 & U_2
\end{pmatrix}$$

where

$$U_1 = B_1$$

$$L_m = A_m U_{m-1}^{-1} \qquad \text{for } m = 2, \ldots n$$

$$U_m = B_m - L_m C_{m-1}$$

Therefore, the system to be solved may be written as

$$L * U * P = F$$

Solving $L * Y = F$ for Y and then solving $U * P = Y$ for P gives the desired pressures.

SECTION V

PROGRAM DESCRIPTIONS

Program DUCT computes the pressure within cylindrical, rectangular, and nonuniform ducts using the finite difference method described in Section IV. Using these values for the pressure, DUCT then calculates the acoustic power and attenuation.

The logical flow of DUCT is represented schematically in Figure 3. A summary of the various subroutines called by DUCT is now given.

DUCT — The main routine which controls the reading of input and the initialization of variables. DUCT calls FUNCT directly or, if the function is to be minimized, calls an optimization routine ZXMIN.

FUNCT — A statement function which does the calling to the subroutines which set up the system to be solved,

solve the system, compute the attenuation of sound, and print the output. By having a statement function do the calling, standard optimization routines can be used which minimize a given function.

ZXMIN — A library subroutine which finds the minimum of a function.

SETUP1 — This subroutine uses a finite difference method to transform the wave equation for a rectangular duct

$$P_{xx} + P_{yy} + \lambda^2 P = 0$$

into centered difference equations at each point. For a point not on the boundary, the equation has the following form:

$$[P_{i+1,j} - 2P_{i,j} + P_{i-1,j}]/Dx^2$$
$$+ [P_{i,j+1} - 2P_{i,j} + P_{i,j-1}]Dy^2 + \lambda^2 P_{i,j} = 0$$

Collecting terms, the equation becomes

$$(2 + 2\frac{Dy^2}{Dx^2} - \lambda^2\,Dy^2)\,P_{i,j} - P_{i,j+1} - P_{i,j+1}$$
$$- \frac{D_y^{\,2}}{D_x^{\,2}}\,P_{i+1,j} - \frac{Dy^2}{Dx^2}\,P_{i+1,j} = 0$$

This system of equations is then written in matrix form.

SETUP — Using the methods and equations presented in Section IV, SETUP produces the finite difference for a nonuniform duct.

SETUP2	This subroutine sets up the matrix for a cylindrical duct as described in Section IV.
BLOCK	To increase the efficiency of the program, the matrix produced in the SETUP routines is stored in block-tridiagonal matrix form. BLOCK calls LU3 to factor the matrix into bidiagonal form. LEQS3 is then called to solve the system using back substitution. BLOCK also invokes CLOSMS, OPENMS, WRITMS and READMS which are system subroutines available on Control Data machines.
OUTPUT	Prints the real and imaginary parts of the pressure calculated in the BLOCK subroutine.
SOUND	Given the pressure, SOUND computes and prints the sound power and attenuation $$dB = 10 \log_{10} (E_2/E_1)$$ where E_1 is the acoustic power at the entrance of the duct and E_2 is the acoustic power at the exit.

Input

The information needed to perform the calculations can be entered through cards or disks. A description of required input is presented here.

Card No.	Columns	Variable Name	Description
1	1-3	IDUCT	An integer representing the duct geometry. Enter -1 for a nonuniform duct, 0 for

Card No.	Columns	Variable Name	Description
			a rectangular duct or 1 for a cylindrical duct. (I3)
2	1-10	XM	Mach Number. For the acoustic linearization to be valid $-.5 \leq XM \leq .5$. Set XM equal to zero for no flow case. (F10.0)
3	1-20	ZETAX	Complex number representing the exit impedance (F10.0, F10.0).
4	1-20	ZETAX	Wall impedance at Y = YMAX. ZETAX = 9999999999 + i9999999999 corresponds to a hard wall (F10.0, F10.0).
5	1-10	XMAX	Length, L, of duct in meters (F10.0).
5	11-20	YMAX	Height, H, of duct in meters. If problem has been made non-dimensional, set YMAX = 1 and XMAX = $\frac{L}{H}$. (F10.0)
6	1-10	ETA	Frequency parameter (F10.0).
7	1-3	IEND	Number of pts in x-direction, excluding pts on the boundaries. Maximum of 100 if subroutines READMS, WRITMS are used. If XM $\neq$ 0, IEND should be odd. (I3)

Card No.	Columns	Variable Name	Description
7	4-6	IEND	Number of pts. in y-direction, excluding pts on boundaries. Maximum of 20. (I3)
8	1-3	KOUT	Controls the form of the output. Set KOUT = 0 to suppress the printing of pressures and KOUT = 1 for complete output. (I3)

If ZXMIN is to be called to find the minimizing values of ZETAY, the following information is required.

Card No.	Columns	Variable Name	Description
8	4-6	NUMMAX	Maximum number of iterations or calls to FUNCT by ZXMIN. (I3)
8	7-16	IH	Number of digits of accuracy desired in the estimates of ZETAY. (I3)

If KOUT = 0, the following card can be left blank.

Card No.	Columns	Variable Name	Description
9	1-3	IOUT	Increment value. The pressure will be printed for pts with abscissas corresponding to the values of I = 1 through IEND in steps of IOUT. (I3)

Card No.	Columns	Variable Name	Description
9	4-6	JOUT	Increment value. The program will print the pressure at pts located on grid lines corresponding to J = 1 through JEND in steps of JOUT. (I3)
9	7-9	IT	The pressure will be printed for pts located on grid lines corresponding to J = 1 through IT. Maximum value is IT = JEND. (Headings will be distorted if JOUT = 1 and IT > 15).
10	1-10	ALPHA	Generating angle in radians of a nonuniform duct. Required only if IDUCT = -1. (F10.0)

Multisectional Linings

If the user desires a multisectional lining, several changes must be made to the program to accommodate the additional impedance parameters (See Section IV). The following changes were used to modify the program for a three-sectional lining in a nonuniform duct. Modifications for rectangular and cylindrical ducts are enclosed in brackets.

Recall that the impedance is a complex number so that an n-sectional lining means ZET, the array containing these values, will be of dimension 2n. To adjust the dimension of ZET, insert the appropriate value in line 7 of DUCT. In DUCT, arrays WA, HH, and G are parameters required by ZXMIN and their dimension depends on the dimension of ZET. Set the

dimension of WA equal to N(N+4). The dimension of HH equal to N(N+1)/2, and the dimension of G equal to N where N is the dimension of ZET (N=2n). For example, for a 3 section lining, DUCT.7 should now read:

```
DIMENSION ZET(6), WA(18), HH(21), G(6)
```

In addition, a dimension statement will need to be inserted after line 8 in SETUP [SETUP1, SETUP2] which reads DIMENSION ZET (6). Also, in line 55 of DUCT, the number following FUNCT in the parameter list is the dimension of ZET, thus it should be changed to

```
ZXMIN (FUNCT, 6, IH, NUMMAX, φ, ZET, HH, G, FMIN, WA, IER)
```

To initialize the values of ZET to those of a uniform lining, the following cards should be inserted after line 51 in DUCT

```
ZET(3) = ZET(1)
ZET(4) = ZET(2)
ZET(5) = ZET(1)
ZET(6) = ZET(2)
```

If the program is restarted, the expressions on the right side of the equations can be replaced with the minimizing values from the previous run.

In order to pass the additional values to the subroutines, lines 10 through 12 in FUNCT should be replaced with

```
IF(IDUCT.LT.0) CALL SETUP (ZET)
IF(IDUCT.EQ.0) CALL SETUP1(ZET)
IF(IDUCT.GT.0) CALL SETUP2(ZET)
```

and the first line of SETUP [SETUP1, SETUP2] should be replaced with

```
SUBROUTINE SETUP (ZET)
[ SUBROUTINE SETUP1(ZET) ]
[ SUBROUTINE SETUP2(ZET) ]
```

To print the values of ZET before each pass, insert these cards after line 9 in FUNCT

```
PRINT 999, (ZET(I), I = 1,6)
999 FORMAT (1X, 6F16.8)
```

where I ranges from 1 to the dimension of ZET.

Finally, the location of the various linings are inserted after SETUP .34 [SETUP1.22, SETUP2.22]. For example, if each lining occupies one third of the duct the following lines are added:

```
ZETAY = ZET(1)+AI * ZET(2)
IF (K.GT.IEND/3)ZETAY=ZET(3)+AI*ZET(4)
IF (K.GT.2*IEND/3) ZETAY=ZET(5)+AI*ZET(6)
```

Radial Modes

If the function to be evaluated is not equal to a constant (one) at the entrance, the pressure at those points must be inputed. Several changes are required in the main program and the SETUP subroutines in order to modify the program for this case.

The following lines should be inserted after line 28 in DUCT,

```
    READ 993 (Y(I), I=1, JEND)
993 FORMAT (12F6.0)
    PRINT 994 (Y(I), I=1, JEND)
994 FORMAT (IX, 16F8.3)
```

In addition, line 47 in SETUP2 and line 51 in SETUP1 should be replaced with

```
10 Y(I) = - AA(I) * Y(I).
```

In SETUP, line 60 should be replaced by

```
10 Y(I) = - A * Y(I).
```

Computations for ZETAX for a Nonuniform Duct

Recall that

$$P = \frac{e^{-2\pi\eta Ri}}{R}$$

and

$$P_R = \frac{-2\pi\eta i P}{Z_x}$$

where η is the frequency and $R = .5e^{-ALPHA \cdot XMAX}$

Therefore

$$Z_x = \frac{-2\pi\eta i P}{P_R}$$

$$= \frac{-2\pi\eta i e^{-2\pi\eta i R}}{R} \qquad \frac{e^{-2\pi\eta Ri}}{R} \quad (\frac{1}{R} + 2\pi\eta i)$$

$$= \frac{R^2\ (2\pi\eta)^2 + 2\pi\eta i R}{1 + (2\pi\eta R)^2}$$

OUTPUT

The output from DUCT consists of the following:

1) The solutions to the wave equation at the indicated points. Note that in reading the printout from left to right, the y-coordinate varies while the x-coordinate is held constant. The last

column of numbers corresponds to the x-coordinate of the points. The headings for the program have been set up for printing a maximum of fifteen points in the y-direction. If desired, the pressure can be printed at addition points, but the headings will be distorted.

2) The computed values for the acoustic power at the entrance and exit of the duct and the resultant attenuation.

3) If ZXMIN is called and a multisectional lining is used, the values of ZET will be printed before the headings for each iteration. When all the iterations have been performed, the minimizing value of ZETAY for the first lining, the attenuation, the convergence criterion, the number of iterations, and IER, an error condition will be printed.

IER = 0 means no errors occurred and convergence has been achieved.

IER = 130 means the iteration was terminated due to excessive rounding errors.

IER = 131 means the iteration was terminated because NUMMAX has been reached.

Section VI

SAMPLE RUNS

In this section, the input and output for a variety of sample cases are presented.

Case I

Cylindrical duct: no flow, uniform lining, pressure printed at every other point in both directions.

Case I Input

```
 1
0.
1.        0.
 .7       -.6
2.          1.
 .5
 50 20
  1
  2  2 20
```

Case II

Conical duct: no flow, uniform lining, pressure printed for first 15 points in y-direction.

Case II Input

```
 -1
 0
.9641     .1861
.3        -.5
1.         1.
1.
 49 20
  1
  2  1 11
  .75
```

Case III

Rectangular duct: no flow, radial modes, pressure printed for first 15 points in y-direction.

Case III	Input	
0		
0.		
1.0453	.332	
.9287	-.7442	
1.	1.	
1.		
50 20		
1		
2 2 20		
-.775 -1.18 -.440 -1.11 ...		Pressures for the
.950 -.075 .990 -.020 ...		20 nodes at the
.170 -.820 -.110 .990 ...		entrance

Case IV

Cylindrical duct: no flow, uniform lining, ZXMIN called to find minimizing value of ZETAY.

Case IV	Input
1	
0.	
1.	0.
.6	-1.5
2.	1.
1.25	
60 20	
0 15 2	
0	

Case V

Rectangular Duct: uniform flow, hard wall, pressure printed at every other point in y-direction.

Case V Input

```
0
.2
1.     0.
999999999999   (20 times)
1.      1.
1.
 25 20
  1
  1  2 20
```

```
ENTER XMAX,YMAX F10.0,F10.0
ENTER ETA F10.0
ENTER IEND(X),JEND(Y) 2I3
50 20
DO YOU WANT COMPLETE OUTPUT? 0 OR 1
ENTER IOUT,JOUT,IT/3I3
```

CASE 1

1 OF 2

CYLINDRICAL DUCT

DX= .0392 DY= .0476 ZETAY= .7000+I -.6000 ZETAX= 1.0000+I 0.0000 ETA= .5000 M= 0.000

REAL PART

Y= .05	Y= .14	Y= .24	Y= .33	Y= .43	Y= .52	Y= .62	Y= .71	Y= .81	Y= .90	X=
-.054	-.050	-.041	-.029	-.014	.002	.017	.030	.037	.038	1.961
-.067	-.063	-.054	-.040	-.023	-.004	.015	.033	.046	.053	1.882
-.084	-.079	-.069	-.053	-.034	-.012	.012	.035	.055	.070	1.804
-.103	-.098	-.087	-.070	-.048	-.022	.006	.035	.063	.087	1.725
-.124	-.119	-.107	-.089	-.065	-.037	-.005	.030	.067	.102	1.647
-.146	-.141	-.129	-.110	-.086	-.056	-.020	.020	.064	.110	1.569
-.167	-.162	-.151	-.133	-.109	-.079	-.041	.003	.053	.109	1.490
-.186	-.182	-.173	-.157	-.135	-.106	-.069	-.023	.031	.094	1.412
-.201	-.198	-.191	-.179	-.161	-.135	-.102	-.058	-.004	.063	1.333
-.209	-.208	-.204	-.196	-.185	-.166	-.140	-.102	-.051	.015	1.255
-.207	-.207	-.208	-.207	-.204	-.196	-.179	-.153	-.111	-.052	1.176
-.191	-.194	-.200	-.207	-.215	-.219	-.218	-.206	-.180	-.135	1.098
-.159	-.164	-.176	-.194	-.214	-.234	-.250	-.259	-.255	-.231	1.020
-.108	-.116	-.135	-.163	-.197	-.234	-.272	-.304	-.327	-.333	.941
-.036	-.047	-.073	-.111	-.160	-.216	-.276	-.336	-.390	-.432	.863
.056	.043	.010	-.038	-.101	-.175	-.257	-.345	-.433	-.516	.784
.169	.153	.115	.058	-.017	-.107	-.211	-.325	-.447	-.571	.706
.298	.280	.239	.175	.091	-.012	-.133	-.270	-.421	-.585	.627
.439	.421	.377	.309	.220	.109	-.023	-.176	-.349	-.545	.549
.585	.567	.523	.455	.365	.252	.117	-.042	-.226	-.441	.471
.727	.709	.668	.604	.518	.411	.281	.127	-.054	-.269	.392
.854	.839	.802	.745	.669	.574	.458	.321	.159	-.035	.314
.955	.943	.913	.868	.806	.729	.635	.525	.397	.248	.235
1.020	1.011	.990	.958	.915	.861	.795	.720	.635	.548	.157
1.038	1.033	1.022	1.006	.983	.956	.922	.884	.845	.820	.078

CASE 1

2 OF 2

IMAGINARY PART

Y= .05	Y= .14	Y= .24	Y= .33	Y= .43	Y= .52	Y= .62	Y= .71	Y= .81	Y= .90	X=
.048	.047	.044	.038	.030	.019	.005	-.012	-.031	-.052	1.961
.035	.034	.033	.031	.027	.021	.011	-.002	-.019	-.040	1.882
.020	.020	.022	.024	.025	.024	.020	.011	-.002	-.022	1.804
.002	.004	.009	.016	.023	.028	.031	.029	.020	.004	1.725
-.020	-.016	-.007	.005	.019	.032	.043	.049	.048	.037	1.647
-.047	-.041	-.028	-.010	.012	.034	.056	.072	.080	.077	1.569
-.080	-.073	-.056	-.031	-.000	.033	.066	.094	.115	.123	1.490
-.122	-.112	-.091	-.059	-.019	.025	.071	.114	.150	.173	1.412
-.172	-.161	-.134	-.095	-.046	.010	.069	.128	.181	.223	1.333
-.231	-.218	-.187	-.141	-.083	-.016	.058	.133	.205	.268	1.255
-.299	-.284	-.249	-.197	-.131	-.053	.034	.125	.217	.303	1.176
-.374	-.357	-.320	-.263	-.190	-.103	-.004	.102	.211	.321	1.098
-.453	-.436	-.396	-.336	-.259	-.165	-.058	.059	.185	.315	1.020
-.533	-.516	-.476	-.415	-.335	-.239	-.128	-.003	.133	.280	.941
-.610	-.594	-.554	-.495	-.416	-.321	-.210	-.085	.055	.211	.863
-.679	-.663	-.626	-.570	-.497	-.407	-.303	-.183	-.048	.106	.784
-.733	-.718	-.685	-.635	-.570	-.490	-.398	-.292	-.172	-.032	.706
-.766	-.754	-.726	-.683	-.629	-.563	-.488	-.404	-.308	-.196	.627
-.774	-.764	-.741	-.708	-.666	-.617	-.564	-.507	-.445	-.374	.549
-.751	-.744	-.727	-.703	-.674	-.643	-.613	-.587	-.566	-.547	.471
-.696	-.691	-.679	-.664	-.647	-.633	-.625	-.630	-.652	-.692	.392
-.607	-.604	-.597	-.589	-.582	-.581	-.592	-.622	-.682	-.782	.314
-.487	-.485	-.482	-.479	-.479	-.486	-.508	-.553	-.639	-.784	.235
-.341	-.340	-.339	-.338	-.341	-.351	-.373	-.419	-.507	-.671	.157
-.176	-.175	-.175	-.175	-.178	-.184	-.198	-.227	-.286	-.412	.078

ACOUSTIC POWER AT ENTRANCE = 1.31163
ACOUSTIC POWER AT EXIT = .00301
ATTENUATION = -26.38735

```
ENTER ETA F10.0
ENTER IEND(X),JEND(Y) 2I3
49 20
DO YOU WANT COMPLETE OUTPUT? 0 OR 1
ENTER IOUT,JOUT,IT/3I3
 ENTER ALPHA,F10.0
```


NON-UNIFORM DUCT
DX= .0200 DY= .0476 ZETAY= .3000+I -.5000 ZETAX= .9641+I .1861 ETA= 1.0000 M= 0.000

REAL PART

Y= .05	Y= .10	Y= .14	Y= .19	Y= .24	Y= .29	Y= .33	Y= .38	Y= .43	Y= .48	Y= .52
-.264	-.262	-.258	-.252	-.244	-.234	-.222	-.206	-.193	-.175	-.156
-.235	-.234	-.230	-.225	-.219	-.211	-.201	-.190	-.178	-.163	-.147
-.199	-.198	-.196	-.192	-.187	-.182	-.175	-.167	-.158	-.147	-.135
-.156	-.156	-.154	-.152	-.150	-.147	-.143	-.139	-.134	-.128	-.120
-.108	-.108	-.108	-.107	-.107	-.107	-.107	-.106	-.106	-.105	-.103
-.055	-.055	-.056	-.058	-.060	-.063	-.066	-.070	-.074	-.076	-.082
.003	.002	-.001	-.004	-.009	-.015	-.021	-.029	-.038	-.046	-.059
.063	.061	.058	.053	.046	.037	.027	.015	.001	-.014	-.031
.126	.124	.119	.112	.103	.092	.078	.062	.044	.023	-.000
.190	.188	.182	.174	.163	.149	.132	.112	.090	.064	.035
.256	.253	.247	.237	.224	.208	.189	.166	.139	.109	.075
.323	.319	.312	.301	.287	.269	.247	.222	.192	.158	.119
.389	.386	.378	.366	.351	.332	.308	.280	.248	.211	.169
.455	.452	.444	.432	.415	.395	.370	.341	.307	.269	.224
.521	.517	.509	.496	.480	.459	.434	.404	.369	.330	.284
.585	.581	.573	.560	.544	.523	.498	.466	.434	.394	.348
.647	.643	.635	.623	.607	.587	.562	.533	.500	.461	.417
.706	.702	.695	.683	.668	.649	.626	.598	.566	.530	.488
.762	.759	.752	.741	.727	.709	.688	.663	.633	.600	.561
.814	.811	.805	.795	.783	.767	.748	.725	.699	.669	.635
.861	.859	.853	.845	.835	.821	.805	.786	.763	.736	.709
.904	.902	.897	.891	.882	.871	.858	.842	.825	.804	.781
.940	.938	.935	.930	.924	.916	.906	.895	.882	.867	.850
.970	.969	.967	.964	.960	.955	.949	.942	.934	.925	.914
.992	.992	.991	.990	.989	.987	.985	.982	.980	.977	.973

IMAGINARY PART

CASE 2

Y= .05	Y= .10	Y= .14	Y= .19	Y= .24	Y= .29	Y= .33	Y= .38	Y= .43	Y= .48	Y= .52
-.177	-.175	-.171	-.164	-.155	-.144	-.131	-.116	-.101	-.083	-.065
-.235	-.232	-.227	-.218	-.208	-.194	-.179	-.161	-.142	-.121	-.098
-.289	-.286	-.280	-.270	-.258	-.243	-.225	-.205	-.182	-.157	-.131
-.339	-.336	-.329	-.318	-.305	-.288	-.268	-.246	-.220	-.193	-.163
-.384	-.380	-.373	-.362	-.347	-.329	-.308	-.284	-.256	-.226	-.193
-.423	-.420	-.412	-.400	-.385	-.366	-.344	-.319	-.290	-.258	-.223
-.457	-.453	-.445	-.433	-.418	-.399	-.376	-.350	-.321	-.288	-.252
-.484	-.480	-.472	-.461	-.445	-.426	-.403	-.378	-.348	-.316	-.280
-.505	-.501	-.494	-.482	-.467	-.448	-.426	-.401	-.372	-.341	-.306
-.519	-.516	-.508	-.497	-.483	-.465	-.444	-.420	-.393	-.363	-.330
-.527	-.524	-.517	-.506	-.493	-.476	-.457	-.435	-.410	-.382	-.352
-.528	-.525	-.519	-.509	-.497	-.482	-.464	-.444	-.422	-.397	-.371
-.523	-.520	-.514	-.506	-.495	-.481	-.466	-.448	-.429	-.408	-.386
-.511	-.509	-.504	-.496	-.487	-.475	-.462	-.447	-.431	-.414	-.397
-.493	-.491	-.487	-.480	-.472	-.463	-.452	-.440	-.427	-.415	-.402
-.469	-.468	-.464	-.459	-.452	-.444	-.436	-.426	-.417	-.409	-.401
-.440	-.438	-.435	-.431	-.426	-.420	-.413	-.407	-.401	-.396	-.393
-.404	-.403	-.401	-.398	-.394	-.389	-.385	-.380	-.377	-.376	-.376
-.364	-.363	-.361	-.359	-.356	-.353	-.350	-.348	-.347	-.348	-.352
-.318	-.318	-.316	-.315	-.313	-.311	-.309	-.308	-.309	-.312	-.318
-.268	-.268	-.267	-.266	-.265	-.263	-.263	-.263	-.265	-.269	-.276
-.214	-.214	-.214	-.213	-.212	-.211	-.211	-.212	-.214	-.219	-.225
-.157	-.157	-.156	-.156	-.155	-.155	-.155	-.156	-.158	-.162	-.168
-.096	-.096	-.096	-.095	-.095	-.095	-.096	-.096	-.098	-.100	-.104
-.033	-.033	-.032	-.032	-.032	-.032	-.032	-.033	-.033	-.034	-.035

ACOUSTIC POWER AT ENTRANCE = .65421
ACOUSTIC POWER AT EXIT = .02541
ATTENUATION = -14.10767

ENTER IOUT,JOUT,IT/3I3

-.775	-1.180	-.440	-1.110	-.110	-.990	.170	-.820	.460	-.640	.620
.760	-.305	.875	-.170	.950	-.075	.990	-.020	.990	-.020	.950
.875	-.170	.760	-.305	.620	-.470	.460	-.640	.170	-.820	-.110
-.440	-1.110	-.775	-1.180							

RECTANGULAR DUCT
DX= .0196 DY= .0476 ZETAY= .9287+I -.7442 ZETAX= 1.0453+I .3320 ETA= 1.0000 M= 0.00

REAL PART

Y= .05	Y= .14	Y= .24	Y= .33	Y= .43	Y= .52	Y= .62	Y= .71	Y= .81	Y= .90	X=
.097	.122	.111	.085	.063	.055	.065	.089	.115	.127	.980
.149	.151	.119	.077	.044	.033	.048	.082	.124	.156	.941
.201	.176	.121	.062	.019	.005	.024	.069	.127	.180	.902
.250	.196	.117	.041	-.012	-.029	-.006	.049	.123	.198	.863
.294	.208	.105	.013	-.048	-.067	-.042	.022	.112	.208	.824
.329	.211	.085	-.021	-.089	-.110	-.081	-.010	.093	.208	.784
.351	.202	.056	-.060	-.133	-.154	-.123	-.046	.066	.198	.745
.358	.180	.019	-.104	-.177	-.197	-.165	-.086	.032	.176	.706
.345	.144	-.026	-.149	-.219	-.237	-.206	-.128	-.008	.142	.667
.311	.094	-.077	-.194	-.257	-.272	-.241	-.168	-.054	.098	.627
.255	.031	-.133	-.237	-.288	-.297	-.270	-.206	-.102	.043	.588
.175	-.045	-.191	-.274	-.308	-.311	-.288	-.238	-.151	-.020	.549
.073	-.131	-.249	-.302	-.315	-.310	-.293	-.261	-.199	-.091	.510
-.049	-.224	-.302	-.319	-.307	-.291	-.283	-.274	-.243	-.165	.471
-.188	-.319	-.348	-.321	-.279	-.254	-.256	-.273	-.279	-.240	.431
-.339	-.412	-.382	-.305	-.232	-.197	-.210	-.258	-.307	-.313	.392
-.494	-.498	-.401	-.271	-.165	-.119	-.146	-.227	-.324	-.381	.353
-.647	-.570	-.401	-.216	-.077	-.022	-.063	-.180	-.327	-.441	.314
-.790	-.624	-.379	-.139	.031	.093	.037	-.116	-.317	-.492	.275
-.913	-.652	-.333	-.043	.155	.222	.151	-.038	-.291	-.529	.235
-1.006	-.652	-.262	.073	.292	.362	.276	.055	-.251	-.554	.196
-1.063	-.617	-.166	.203	.437	.507	.408	.159	-.195	-.564	.157
-1.073	-.546	-.045	.344	.583	.650	.540	.271	-.124	-.560	.118
-1.032	-.437	.099	.490	.723	.783	.667	.398	-.038	-.541	.078
-.935	-.291	.265	.632	.848	.899	.781	.507	.062	-.503	.039

IMAGINARY PART

Y= .05	Y= .14	Y= .24	Y= .33	Y= .43	Y= .52	Y= .62	Y= .71	Y= .81	Y= .90	X=
-.203	-.101	-.013	.053	.092	.103	.088	.048	-.016	-.099	.980
-.190	-.078	.012	.075	.111	.121	.108	.071	.009	-.075	.941
-.164	-.046	.041	.098	.127	.136	.124	.093	.038	-.043	.902
-.125	-.006	.073	.119	.140	.145	.137	.114	.070	-.004	.863
-.072	.041	.106	.137	.147	.148	.144	.132	.101	.041	.824
-.006	.093	.139	.150	.146	.142	.143	.144	.132	.090	.784
.071	.150	.169	.156	.136	.127	.134	.151	.160	.141	.745
.157	.207	.194	.154	.117	.101	.115	.149	.183	.192	.706
.249	.261	.213	.143	.086	.064	.085	.138	.200	.240	.667
.342	.311	.222	.120	.044	.016	.044	.117	.208	.283	.627
.432	.351	.219	.086	-.010	-.043	-.007	.085	.206	.317	.588
.513	.379	.203	.039	-.073	-.110	-.067	.044	.193	.340	.549
.580	.390	.173	-.018	-.145	-.185	-.135	-.007	.169	.351	.510
.627	.382	.128	-.085	-.223	-.264	-.208	-.065	.134	.347	.471
.648	.353	.068	-.161	-.303	-.345	-.282	-.129	.088	.328	.431
.638	.300	-.006	-.241	-.383	-.421	-.356	-.196	.033	.294	.392
.594	.224	-.091	-.322	-.457	-.490	-.424	-.264	-.030	.244	.353
.512	.124	-.185	-.400	-.520	-.547	-.482	-.328	-.097	.180	.314
.393	.003	-.284	-.470	-.567	-.585	-.526	-.385	-.167	.104	.275
.236	-.135	-.383	-.527	-.593	-.601	-.552	-.433	-.235	.018	.235
.045	-.286	-.476	-.565	-.594	-.590	-.557	-.468	-.298	-.076	.196
-.176	-.444	-.558	-.578	-.563	-.548	-.537	-.489	-.352	-.177	.157
-.419	-.601	-.622	-.563	-.497	-.472	-.490	-.497	-.392	-.285	.118
-.674	-.750	-.661	-.514	-.394	-.360	-.415	-.495	-.421	-.409	.078
-.932	-.882	-.669	-.428	-.253	-.209	-.309	-.487	-.481	-.602	.039

ACOUSTIC POWER AT ENTRANCE = .07034
ACOUSTIC POWER AT EXIT = .00193
ATTENUATION = -15.60759

CASE 4

1 OF 4

```
ENTER DUCT GEOM\ NONUNIFORM,RECT,CYLIND;-1,0,1
ENTER MACH NO. F10.0
ENTER ZETAX F10.0,F10.0
ENTER ZETAY F10.0,F10.0
ENTER XMAX,YMAX F10.0,F10.0
ENTER ETA F10.0
ENTER IEND(X),JEND(Y) 2I3
60 20
DO YOU WANT COMPLETE OUTPUT? 0 OR 1
ENTER IOUT,JOUT,IT/3I3


CYLINDRICAL DUCT
DX= .0328 DY= .0476  ZETAY=  .6000+I-1.5000  ZETAX= 1.0000+I 0.0000  ETA= 1.2500  M= 0.000


ACOUSTIC POWER AT ENTRANCE =    2.66222
ACOUSTIC POWER AT EXIT =     .64423
ATTENUATION =   -5.16201
```

```
CYLINDRICAL DUCT
DX= .0328 DY= .0475   ZETAY=  .6000+I-1.5000   ZETAX= 1.0000+I 0.0000   ETA= 1.2500   M= 0.000



ACOUSTIC POWER AT ENTRANCE =     2.66222
ACOUSTIC POWER AT EXIT =      .64423
ATTENUATION =   -6.16201



CYLINDRICAL DUCT
DX= .0328 DY= .0475   ZETAY=  .6000+I-1.5000   ZETAX= 1.0000+I 0.0000   ETA= 1.2500   M= 0.000



ACOUSTIC POWER AT ENTRANCE =     2.66222
ACOUSTIC POWER AT EXIT =      .64423
ATTENUATION =   -6.16201



CYLINDRICAL DUCT
DX= .0328 DY= .0475   ZETAY= 1.0129+I-1.4825   ZETAX= 1.0000+I 0.0000   ETA= 1.2500   M= 0.000



ACOUSTIC POWER AT ENTRANCE =     2.60974
ACOUSTIC POWER AT EXIT =      .67955
ATTENUATION =   -5.84377
```

CASE 4

3 OF 4

```
CYLINDRICAL DUCT
DX= .0328 DY= .0476  ZETAY=  .8065+I-1.4912  ZETAX= 1.0000+I 0.0000  ETA= 1.2500  M= 0.000



ACOUSTIC POWER AT ENTRANCE =    2.63518
ACOUSTIC POWER AT EXIT =     .64948
ATTENUATION =   -6.08246



CYLINDRICAL DUCT
DX= .0328 DY= .0476  ZETAY=  .6206+I-1.4991  ZETAX= 1.0000+I 0.0000  ETA= 1.2500  M= 0.000



ACOUSTIC POWER AT ENTRANCE =    2.65959
ACOUSTIC POWER AT EXIT =     .64267
ATTENUATION =   -6.16827



CYLINDRICAL DUCT
DX= .0328 DY= .0476  ZETAY=  .6206+I-1.4991  ZETAX= 1.0000+I 0.0000  ETA= 1.2500  M= 0.000



ACOUSTIC POWER AT ENTRANCE =    2.65959
ACOUSTIC POWER AT EXIT =     .64267
ATTENUATION =   -6.16527
```

CASE 4


```
CYLINDRICAL DUCT
DX= .0328 DY= .0475  ZETAY=  .6205+I-1.4991  ZETAX= 1.0000+I 0.0000  ETA= 1.2500  M= 0.000


ACOUSTIC POWER AT ENTRANCE =    2.65959
ACOUSTIC POWER AT EXIT =      .64267
ATTENUATION =   -6.16827


CYLINDRICAL DUCT
DX= .0328 DY= .0476  ZETAY=  .6568+I-1.5548  ZETAX= 1.0000+I 0.0000  ETA= 1.2500  M= 0.000


ACOUSTIC POWER AT ENTRANCE =    2.65670
ACOUSTIC POWER AT EXIT =      .64260
ATTENUATION =   -6.16400


CYLINDRICAL DUCT
DX= .0328 DY= .0475  ZETAY=  .6387+I-1.5270  ZETAX= 1.0000+I 0.0000  ETA= 1.2500  M= 0.000


ACOUSTIC POWER AT ENTRANCE =    2.65797
ACOUSTIC POWER AT EXIT =      .54199
ATTENUATION =   -6.17025
```

CASE 5

1 OF 2

```
ENTER IEND(X),JEND(Y) 2I3
25 20
DO YOU WANT COMPLETE OUTPUT? 0 OR 1
ENTER IOUT,JOUT,IT/3I3
```

```
RECTANGULAR DUCT
DX= .0385 DY= .0476   ZETAY=*******+I*******   ZETAX= 1.2000+I 0.0000   ETA= 1.0000   M=  .200
```

REAL PART

Y= .05	Y= .14	Y= .24	Y= .33	Y= .43	Y= .52	Y= .62	Y= .71	Y= .81	Y= .90	X=
.330	.330	.330	.330	.330	.330	.330	.330	.330	.330	.962
.133	.133	.133	.133	.133	.133	.133	.133	.133	.133	.923
-.069	-.069	-.069	-.069	-.069	-.069	-.069	-.069	-.069	-.069	.885
-.269	-.269	-.269	-.269	-.269	-.269	-.269	-.269	-.269	-.269	.846
-.457	-.457	-.457	-.457	-.457	-.457	-.457	-.457	-.457	-.457	.808
-.626	-.626	-.626	-.626	-.626	-.626	-.626	-.626	-.626	-.626	.769
-.771	-.771	-.771	-.771	-.771	-.771	-.771	-.771	-.771	-.771	.731
-.883	-.883	-.883	-.883	-.883	-.883	-.883	-.883	-.883	-.883	.692
-.960	-.960	-.960	-.960	-.960	-.960	-.960	-.960	-.960	-.960	.654
-.998	-.998	-.998	-.998	-.998	-.998	-.998	-.998	-.998	-.998	.615
-.995	-.995	-.995	-.995	-.995	-.995	-.995	-.995	-.995	-.995	.577
-.952	-.952	-.952	-.952	-.952	-.952	-.952	-.952	-.952	-.952	.538
-.870	-.870	-.870	-.870	-.870	-.870	-.870	-.870	-.870	-.870	.500
-.753	-.753	-.753	-.753	-.753	-.753	-.753	-.753	-.753	-.753	.462
-.605	-.605	-.605	-.605	-.605	-.605	-.605	-.605	-.605	-.605	.423
-.433	-.433	-.433	-.433	-.433	-.433	-.433	-.433	-.433	-.433	.385
-.243	-.243	-.243	-.243	-.243	-.243	-.243	-.243	-.243	-.243	.346
-.044	-.044	-.044	-.044	-.044	-.044	-.044	-.044	-.044	-.044	.308
.158	.158	.158	.158	.158	.158	.158	.158	.158	.156	.269
.352	.352	.352	.352	.352	.352	.352	.352	.352	.352	.231
.533	.533	.533	.533	.533	.533	.533	.533	.533	.533	.192
.692	.692	.692	.692	.692	.692	.692	.692	.692	.692	.154
.822	.822	.822	.822	.822	.822	.822	.822	.822	.822	.115
.920	.920	.920	.920	.920	.920	.920	.920	.920	.920	.077
.980	.980	.980	.980	.980	.980	.980	.980	.980	.980	.038

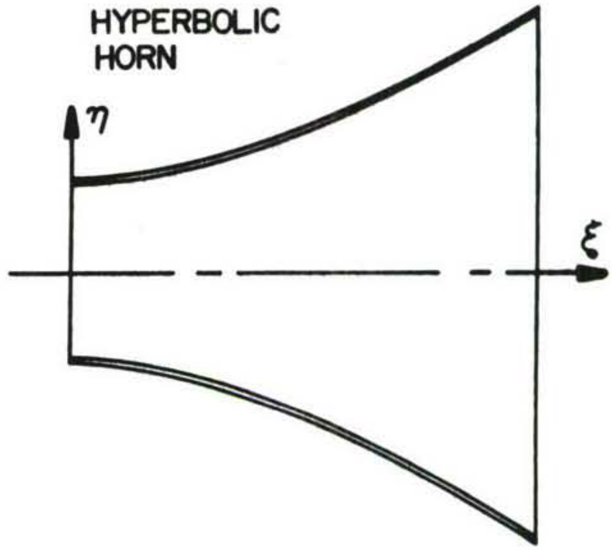
IMAGINARY PART

Y= .05	Y= .14	Y= .24	Y= .33	Y= .43	Y= .52	Y= .62	Y= .71	Y= .81	Y= .90	X=
.944	.944	.944	.944	.944	.944	.944	.944	.944	.944	.962
.991	.991	.991	.991	.991	.991	.991	.991	.991	.991	.923
.998	.998	.998	.998	.998	.998	.998	.998	.998	.998	.885
.965	.965	.965	.965	.965	.965	.965	.965	.965	.965	.846
.892	.892	.892	.892	.892	.892	.892	.892	.892	.892	.808
.783	.783	.783	.783	.783	.783	.783	.783	.783	.783	.769
.642	.642	.642	.642	.642	.642	.642	.642	.642	.642	.731
.476	.476	.476	.476	.476	.476	.476	.476	.476	.476	.692
.289	.289	.289	.289	.289	.289	.289	.289	.289	.289	.654
.091	.091	.091	.091	.091	.091	.091	.091	.091	.091	.615
-.111	-.111	-.111	-.111	-.111	-.111	-.111	-.111	-.111	-.111	.577
-.308	-.308	-.308	-.308	-.308	-.308	-.308	-.308	-.308	-.308	.538
-.493	-.493	-.493	-.493	-.493	-.493	-.493	-.493	-.493	-.493	.500
-.659	-.659	-.659	-.659	-.659	-.659	-.659	-.659	-.659	-.659	.462
-.797	-.797	-.797	-.797	-.797	-.797	-.797	-.797	-.797	-.797	.423
-.903	-.903	-.903	-.903	-.903	-.903	-.903	-.903	-.903	-.903	.385
-.972	-.972	-.972	-.972	-.972	-.972	-.972	-.972	-.972	-.972	.346
-1.002	-1.002	-1.002	-1.002	-1.002	-1.002	-1.002	-1.002	-1.002	-1.002	.308
-.991	-.991	-.991	-.991	-.991	-.991	-.991	-.991	-.991	-.991	.269
-.939	-.939	-.939	-.939	-.939	-.939	-.939	-.939	-.939	-.939	.231
-.850	-.850	-.850	-.850	-.850	-.850	-.850	-.850	-.850	-.850	.192
-.725	-.725	-.725	-.725	-.725	-.725	-.725	-.725	-.725	-.725	.154
-.572	-.572	-.572	-.572	-.572	-.572	-.572	-.572	-.572	-.572	.115
-.395	-.395	-.395	-.395	-.395	-.395	-.395	-.395	-.395	-.395	.077
-.201	-.201	-.201	-.201	-.201	-.201	-.201	-.201	-.201	-.201	.038

ACOUSTIC POWER AT ENTRANCE = 1.12897
ACOUSTIC POWER AT EXIT = 1.14285
ATTENUATION = .05309

GOVERNING EQ.

$$\frac{\partial^2 P}{\partial \xi^2} + \frac{\partial^2 P}{\partial \eta^2} + (2\pi\lambda)^2 P = 0$$



TRANSFORMS VIA

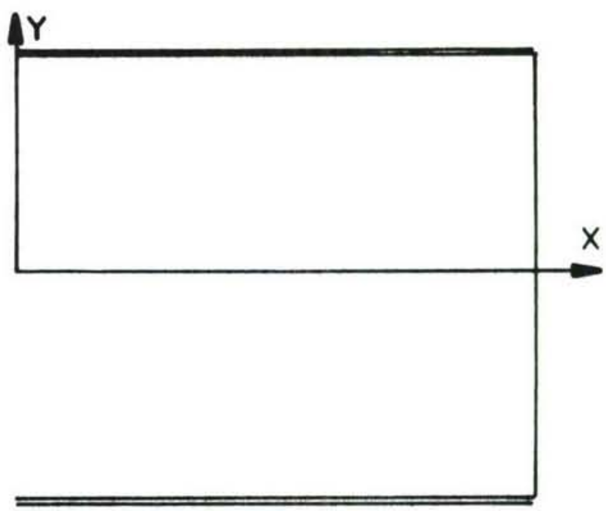
$$\xi = \frac{e^X - e^{-X}}{2} \cos y$$

$$\eta = \frac{e^X + e^{-X}}{2} \sin y$$

TO

AND THE GOVERNING EQ.
BECOMES

$$\frac{\partial^2 P}{\partial X^2} + \frac{\partial^2 P}{\partial Y^2} + (2\pi\lambda)^2 \cdot F(X,Y) \cdot P = 0$$

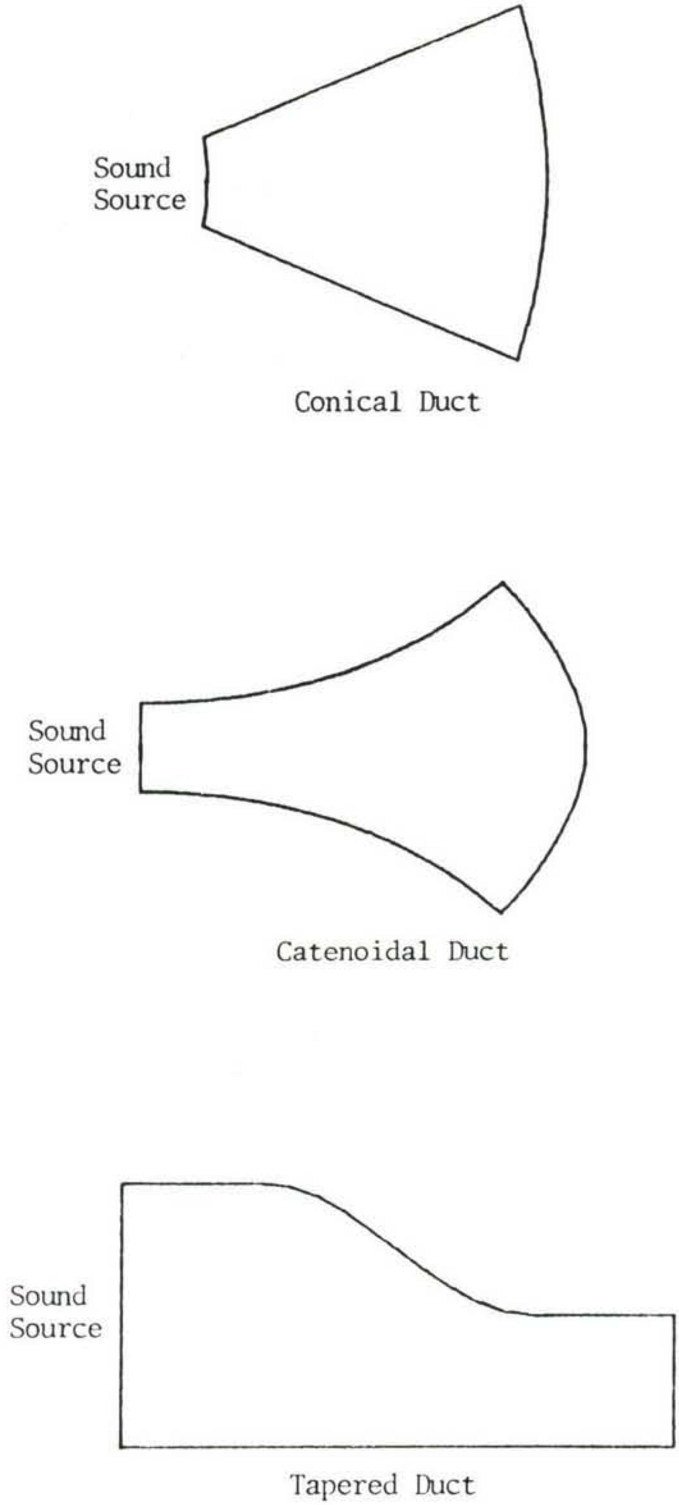


WHERE FOR THE
HYBERBOLIC HORN

$$F(X,Y) = \frac{e^{2X} - e^{-2X} + 2(\cos^2 y - \sin^2 y)}{4}$$

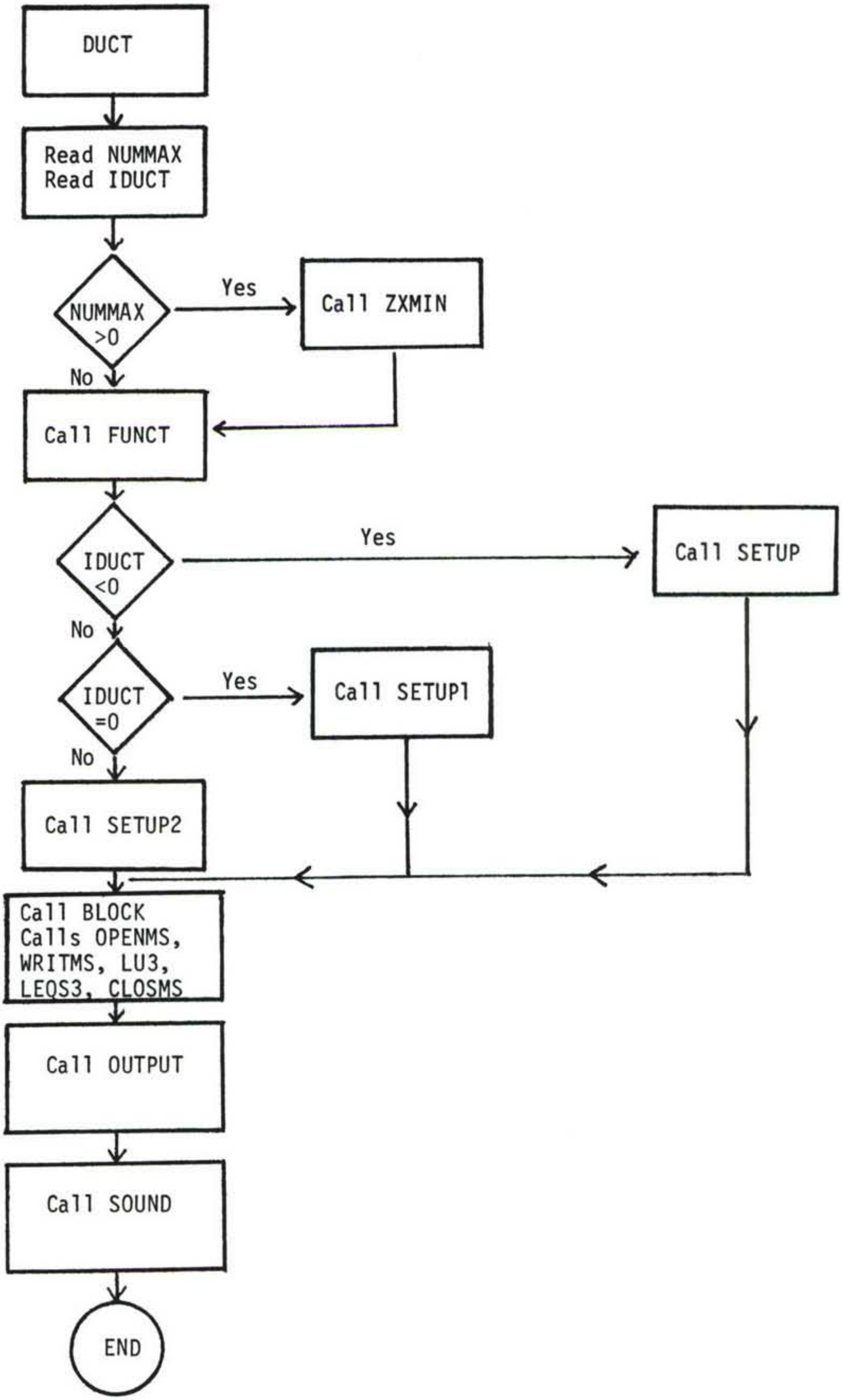
Example of conformal map.

FIGURE 1



Examples of Variable Area Ducts

FIGURE 2



Logical Flow of Duct

FIGURE 3

APPENDIX

```
      PROGRAM DUCT( INPUT,OUTPUT, TAPE 1, TAPE 2, TAPE 3, TAPE 4 )
      COMMON/BIG/AI,ZETAX,ZETAY,XMAX,YMAX,ETA,XM,IEND,JEND,DYDX,M,NN,
     1CON,A,BBBB,B(20,20),C(20,20),T(20,20),D,Y(20),IP(20),V(20),DX,DY,
     2 AA(20),OUT(20),IOUT,JOUT,KOUT,IT,IDUCT,KEND,DB
      COMPLEX B,C,T,D,Y,V,AI,ZETAX,ZETAY ,C1, A,A1,B1,AA
      EXTERNAL FUNCT
      DIMENSION ZET(2),G(2),HH(3),WA(6)
       M=20
       PRINT 981
       READ 983, IDUCT
981   FORMAT(* ENTER DUCT GEOM\ NONUNIFORM,RECT,CYLIND;-1,0,1 *)
      PRINT 991
      READ 990, XM
      PRINT 988
      READ 990, ZETAX
      PRINT 987
      READ 990, ZETAY
      PRINT 986
      READ 990, XMAX, YMAX
      PRINT 985
      READ 990, ETA
      PRINT 984
      READ 992,IEND, JEND
      PRINT 992,IEND,JEND
      PRINT 982
      READ 992,KOUT,NUMMAX,IH
      PRINT 989
      READ 992, IOUT,JOUT,IT
989   FORMAT(1X,*ENTER IOUT,JOUT,IT/3I3*)
992   FORMAT(3I3)
982   FORMAT(* DO YOU WANT COMPLETE OUTPUT? 0 OR 1 *)
983   FORMAT(3I3,F10.0)
984   FORMAT(* ENTER IEND(X),JEND(Y) 2I3 *)
985   FORMAT(* ENTER ETA F10.0 *)
986   FORMAT(* ENTER XMAX,YMAX F10.0,F10.0 *)
987   FORMAT(* ENTER ZETAY F10.0,F10.0 *)
988   FORMAT(* ENTER ZETAX F10.0,F10.0 *)
990   FORMAT(2F10.0)
991   FORMAT(* ENTER MACH NO. F10.0 *)
      AI=(0.,1.)
      ZETAX=ZETAX*(1.+XM)
      NN=JEND
      DX=XMAX/(IEND+1)
      DY=YMAX/(JEND+1)
      DYDX=DY/DX
      CON=2.*3.14159265358979*ETA
  1    A=-(1.-XM*XM)*DYDX**2-CON*XM*AI*DYDX*DY
      BBBB=2.+2.*DYDX**2-CON**2*DY**2
      BBBB=BBBB-2.*XM*XM*DYDX**2
      ZET (1)=REAL(ZETAY)
      ZET (2)=AIMAG(ZETAY)
```

```
      IF(NUMMAX.GT.0) GO TO 100
      CALL FUNCT(2,ZET,FMIN)
      GO TO 500
  100 NVAR=2
      IOPT=0
      CALL ZXMIN(FUNCT,NVAR,IH,NUMMAX,IOPT,ZET,HH,G,FMIN,WA,IER)
      PRINT 999,(ZET(I),I=1,2),FMIN,IH,NUMMAX,IER
999   FORMAT(1X,3F16.8,3I5)
500   STOP
      END
```

```
      SUBROUTINE SETUP
C
C CYLINDRICAL NON-UNIFORM DUCT USING CONFORMAL MAP
C
       COMMON/BIG/AI,ZETAX,ZETAY,XMAX,YMAX,ETA,XM,IEND,JEND,DYDX,M,NN,
     1CON,A,BBBB,B(20,20),C(20,20),T(20,20),D,Y(20),IP(20),V(20),DX,DY,
     2 AA(20),OUT(20),IOUT,JOUT,KOUT,IT,IDUCT,KEND,DB
      COMPLEX B,C,T,D,Y,V,AI,ZETAX,ZETAY ,C1, A,A1,B1,AA
      REWIND 1
      PRINT 999
      READ 998, ALPHA
  998 FORMAT(2F10.0)
  999 FORMAT(1X,* ENTER ALPHA,F10.0 *)
      A=-DYDX**2+ALPHA*DY*DYDX/2.
      DO 100 J=1,JEND
100   AA(J)=A
      DO 6 K=1,IEND
      DO 5 I=1,JEND
      C(I,I)=-DYDX**2-ALPHA*DY*DYDX/2.
      DO 5 J=1,JEND
      IF(J.NE.I) C(   I,J)=0.
      B(   I, J)=0.
      XK=.5
      ARG1=DX*FLOAT(K)
      ARG2=ALPHA*FLOAT(I)*DY
      FOFY=-COS( ARG2)/SIN(ARG2)
      SHAPE=ALPHA*ALPHA*EXP(2.*ALPHA*ARG1)*XK*XK
      IF(I.EQ.J) B(I,J)=BBBB + (1.-SHAPE)*CON**2*DY**2
      IF(J .EQ. I-1) B(I,J)=-1.-ALPHA*FOFY*DY/2.
      IF(J .EQ. I+1) B(I,J)=-1.+ALPHA*FOFY*DY/2.
    5 CONTINUE
C
C  INCLUDE BOUNDARY DATA CORRESPONDING TO Y=0 AND Y=1
C
      B1=-2.*CON*AI*DY*ALPHA*XK*EXP(ALPHA*ARG1)/ZETAY
      B(JEND,JEND-1)=-2.
      B(1,2)=-2.
      ARG2=ALPHA*FLOAT(JEND)*DY
      FOFY=-COS(ARG2)/SIN(ARG2)
      B(JEND,JEND)=B(JEND,JEND)+(ALPHA*FOFY*DY/2. - 1.)*B1
      IF(K .EQ. IEND) GO TO 6
      WRITE(1) AA,B,C
    6 CONTINUE
C
C  INCLUDE BOUNDARY DATA CORRESPONDING TO X=1
C
      SHAPE=ALPHA*XK*EXP(ALPHA*ARG1)
      CON1=CON
      CON=CON*SHAPE
      DO 7 I=1,JEND
    7 B(I,I)=B(I,I)-C(I,I)*2.*CON*AI*DX/ZETAX
      CON=CON1
      WRITE(1) AA,B,C
      DO 8 J=1,JEND
    8 AA(J)=A
      DO 10 I=1,JEND
   10 Y(I)=-A
      REWIND 1
      RETURN
      END
```

```
      SUBROUTINE BLOCK
      COMMON/BIG/AI,ZETAX,ZETAY,XMAX,YMAX,ETA,XM,IEND,JEND,DYDX,M,NN,
     1CON,A,BBBB,B(20,20),C(20,20),T(20,20),D,Y(20),IP(20),V(20),DX,DY,
     2 AA(20),OUT(20),IOUT,JOUT,KOUT,IT,IDUCT,KEND,DB
      COMPLEX B,C,T,D,Y,V,AI,ZETAX,ZETAY ,C1, A,A1,B1,AA,CC
      DIMENSION CC(20,20)
      DIMENSION INDEX(301)
      CALL OPENMS(2,INDEX,301,0)
      REWIND 1
      REWIND 3
      REWIND 4
      READ(4)   (Y(I),I=1,JEND)
      READ(1) AA,B,CC
      CALL WRITMS(2,B,800,1,-1,0)
      CALL WRITMS(2,CC,800,2,-1,0)
      CALL WRITMS(2,Y,40,3,-1,0)
      DO 100 K=2,IEND
      DO 12 I=1,JEND
   12 V(I)=Y(I)
      READ(4)   (Y(I),I=1,JEND)
      DO 15 I=1,JEND
      DO 15 J=1,JEND
      C(I,J)=CC(I,J)
   15 T(I,J)=B(I,J)
      CALL LU3(NN,T,M,IP)
      CALL LEQS3(NN,T,M,IP,V)
      DO 20 I=1,JEND
      A=AA(I)
      IF(K .EQ. IEND)                 A = A + C(I,I)
       D=Y(I)
   20 Y(I)=D-A*V(I)
      READ(1) AA,B,CC
      DO 50 I=1,JEND
      DO 30 J=1,JEND
   30 V(J)=C(J,I)
      CALL LEQS3(NN,T,M,IP,V)
      DO 40 J=1,JEND
      A=AA(J)
      IF(K .EQ. IEND)                 A = A + C(J,J)
   40 B(J,I)=B(J,I)-A*V(J)
   50 CONTINUE
      KK=3*K-3
      CALL WRITMS(2,B,800,KK+1,-1,0)
      CALL WRITMS(2,C,800,KK+2,-1,0)
      CALL WRITMS(2,Y,40,KK+3,-1,0)
  100 CONTINUE
```

```
C
C  INVERT UPPER TRIANGULAR MATRIX TO OBTAIN SOLUTION
C
      DO 110 I=1,JEND
      V(I)=Y(I)
      DO 110 J=1,JEND
  110 T(I,J)=B(I,J)
      CALL LU3(NN,T,M,IP)
      CALL LEQS3(NN,T,M,IP,V)
      DO 120 I=1,JEND
  120 Y(I)=V(I)
      REWIND 1
      WRITE(3) (Y(I),I=1,JEND)
      IEND1=IEND-1
      DO 200 K=1,IEND1
      KK=3*IEND-3*K-3
      CALL READMS(2,B,800,KK+1)
      CALL READMS(2,C,800,KK+2)
      CALL READMS(2,AA,40,KK+3)
      DO 140 I=1,JEND
       V(I)=0.
      DO 130 J=1,JEND
  130 V(I)=C(I,J)*Y(J)+V(I)
  140 V(I)=AA(I)-V(I)
      CALL LU3(NN,B,M,IP)
      CALL LEQS3(NN,B,M,IP,V)
      DO 160 I=1,JEND
  160  Y(I)=V(I)
      WRITE(3) (Y(I),I=1,JEND)
  200 CONTINUE
      REWIND 1
       CALL CLOSMS(2)
      RETURN
      END
```

```
      SUBROUTINE OUTPUT
       COMMON/BIG/AI,ZETAX,ZETAY,XMAX,YMAX,ETA,XM,IEND,JEND,DYDX,M,NN,
     1CON,A,BBBB,B(20,20),C(20,20),T(20,20),D,Y(20),IP(20),V(20),DX,DY,
     2 AA(20),OUT(20),IOUT,JOUT,KOUT,IT,IDUCT,KEND,DB
      COMPLEX B,C,T,D,Y,V,AI,ZETAX,ZETAY ,C1, A,A1,B1,AA
      PRINT 998
      DATA YEQ/3H Y=/,XEQ/3H X=/
      IF(IDUCT .GT. 0) PRINT 993
      IF(IDUCT .EQ. 0 ) PRINT 992
      IF(IDUCT .LT. 0 ) PRINT 991
  991 FORMAT(* NON-UNIFORM DUCT *)
  992 FORMAT(* RECTANGULAR DUCT *)
  993 FORMAT(* CYLINDRICAL DUCT *)
      PRINT 996, DX, DY, ZETAY, ZETAX, ETA ,XM
      IF(KOUT .EQ. 0) RETURN
      PRINT 989
      REWIND 3
      DO 200 J=1,IT,JOUT
200   OUT(J)=J*DY
      PRINT 990,((YEQ,OUT(J)),J=1,IT,JOUT),XEQ
990   FORMAT(1X,15(1X,A3,F4.2),3X,A3)
      L=0
      DO 210 I=1,IEND
      READ(3) (Y(J),J=1,JEND)
      L=L+1
      IF(L .LT. IOUT .AND. I .GT. 1) GO TO 210
      DO 209 J=1,JEND,JOUT
  209 OUT(J)=REAL(Y(J))
      X=(IEND-I+1)*DX
       PRINT 999,(OUT(J),J=1,IT,JOUT),X
      L=0
  210 CONTINUE
      REWIND 3
      PRINT 998
      PRINT 995
      PRINT 998
      DO 201 J=1,IT,JOUT
201   OUT(J)=J*DY
      PRINT 990,((YEQ,OUT(J)),J=1,IT,JOUT),XEQ
      L=0
      DO 212 I=1,IEND
      READ(3) (Y(J),J=1,JEND)
      L=L+1
      IF(L .LT. IOUT .AND. I .GT. 1) GO TO 212
      DO 211 J=1,JEND,JOUT
  211 OUT(J)=AIMAG(Y(J))
      X=(IEND-I+1)*DX
       PRINT 999,(OUT(J),J=1,IT,JOUT),X
      L=0
  212 CONTINUE
```

```
  934 FORMAT(1X,12F10.5)
  995 FORMAT( 20X, *IMAGINARY PART*)
  996 FORMAT(1X,*DX=*,F6.4,* DY=*,F6.4,*  ZETAY=*,F7.4,*+I*,F7.4,
     1 *  ZETAX=*,F7.4,*+I*,F7.4,*  ETA=*,F7.4,*  M=*,F7.4,//)
989   FORMAT(20X,*REAL PART*,//)
  997 FORMAT(1H1)
  998 FORMAT(1X,///)
  999 FORMAT(1X,16F8.3)
      RETURN
      END
```

```
      SUBROUTINE SOUND
       COMMON/BIG/AI,ZETAX,ZETAY,XMAX,YMAX,ETA,XM,IEND,JEND,DYDX,M,NN,
     1CON,A,BBBB,B(20,20),C(20,20),T(20,20),D,Y(20),IP(20),V(20),DX,DY,
     2 AA(20),OUT(20),IOUT,JOUT,KOUT,IT,IDUCT,KEND,DB
      COMPLEX B,C,T,D,Y,V,AI,ZETAX,ZETAY ,C1, A,A1,B1,AA
      REWIND 3
      DA=DY
      IF ( XM .NE. 0. ) GO TO 299
      SUM1=0.
      READ(3) (Y(I),I=1,JEND)
      READ(3) (V(I),I=1,JEND)
      DO 220 J=1,JEND
      IF(IDUCT .NE. 0) DA=J*DY
  220 SUM1=SUM1+REAL(AI*CONJG(Y(J))*(Y(J)-V(J)))
     1 *DA
      SUM2=0.
      KLAST=IEND-3
      DO 225 K=1,KLAST
      READ(3) (Y(I),I=1,JEND)
  225 CONTINUE
      READ(3) (V(I),I=1,JEND)
      DO 230 J=1,JEND
      IF(IDUCT .NE. 0) DA=J*DY
  230 SUM2=SUM2+REAL(AI*CONJG(V(J))*(Y(J)-V(J)))
     1 *DA
      PRINT 998
      PRINT 994,SUM2,SUM1
      IF(SUM2.LT.0.) RETURN
      DB=10.*ALOG10(SUM1/SUM2)
      PRINT 999,DB
      RETURN
  299 IEND2=IEND-2
      READ(3) (Y(J),J=1,JEND)
      READ(3) (V(J),J=1,JEND)
       X=XMAX-DX
       DO 300 J=1,JEND
      T(1,J)=Y(J)
  300 B(1,J)=CEXP(CON*AI*X/XM)*(Y(J)-V(J))/DX
      DO 400 I=1,IEND2
      FACT=FLOAT(I)-2.*FLOAT(I/2)
      WT=4.
      IF(FACT .LT. .5) WT=2.
       X=XMAX-DX-DX*FLOAT(I)
      READ(3) (AA(J),J=1,JEND)
      DO 350 J=1,JEND
      C1=(Y(J)-AA(J))/(2.*DX)
      B(1,J)=B(1,J)+WT*C1*CEXP(CON*AI*X/XM )
      Y(J)=V(J)
  350 V(J)=AA(J)
  400 CONTINUE
```

```
      DO 450 J=1,JEND
      C1=(Y(J)-AA(J))/DX
      B(1,J)=B(1,J) +CEXP(CON*AI*DX/XM)*C1
       Y(J)=T(1,J)
      B1=DX*B(1,J)/3.
      A1=CON*B1/XM
      X=XMAX-DX
      C1=CON*Y(J)*CEXP(CON*AI*X/XM)
      B(1,J)=(A1+C1)*CEXP(-CON*AI*DX/XM)
  450 CONTINUE
      C1=CON*AI/(XM*XM+XM)
      X=XMAX-DX
      B1=-XM*CEXP(C1*X)+XM*CEXP(C1*DX)
      IF(NT.EQ.2.) PRINT 993
  993 FORMAT(* IEND IS EVEN YIELDING ERROR IN QUADRATURE*)
      SUM2=0.
      DO 500 J=1,JEND
      IF(J.EQ.1) A1=(Y(J+1)-Y(J))/DY
      IF(J .EQ. JEND) A1=(Y(J)-Y(J-1))/DY
      IF(J .GT. 1 .AND. J .LT. JEND) A1=(Y(J+1)-Y(J-1))/(2.*DY)
       XI=(1.+XM)*CONJG(Y(J))*Y(J) + XM*(1.+XM)*(1.+XM)*A1*CONJG(A1)/
     1 (2.*CON*CON)
      IF( IDUCT .NE. 0) DA=J*DY
      SUM2=SUM2+DA*XI
  500 CONTINUE
      SUM1=0.
      DO 600 J=1,JEND
      IF(J.EQ.1) A1=(AA(J+1)-AA(J))/DY
      IF(J .EQ. JEND) A1=(AA(J)-AA(J-1))/DY
      IF(J .GT. 1 .AND. J .LT. JEND) A1=(AA(J+1)-AA(J-1))/(2.*DY)
      IF(J .EQ. 1) B1=(B(1,2)-B(1,1))/DY
      IF(J .EQ. JEND) B1=(B(1,JEND)-B(1,JEND-1))/DY
      IF(J .GT. 1 .AND. J .LT. JEND) B1=(B(1,J+1)-B(1,J-1))/(2.*DY)
      C1=AI*A1+AI*XM*B1/CON
      XI=REAL(CONJG(AA(J))*B(1,J)/CON)
      XI=XI+XM*CONJG(AA(J))*AA(J)/2.
      XI=XI+XM*(CONJG(B(1,J))*B(1,J)+CONJG(C1)*C1) / (2.*CON**2)
      IF( IDUCT .NE. 0) DA=J*DY
      SUM1=SUM1+DA*XI
  600 CONTINUE
      PRINT 998
      PRINT 994,SUM1,SUM2
      IF(SUM1 .LT. 0.) RETURN
      DB=10.*ALOG10(SUM2/SUM1)
      PRINT 999,DB
994   FORMAT(1X,*ACOUSTIC POWER AT ENTRANCE = *,F10.5,/,
     1 1X,*ACOUSTIC POWER AT EXIT = *,F10.5)
  997 FORMAT(1H1)
  998 FORMAT(1X,///)
999   FORMAT(1X,*ATTENUATION = *,F10.5)
      RETURN
      END
```

```
      SUBROUTINE SETUP1
C
C THIS SUBROUTINE SETS-UP THE FINITE DIFFERENCE MATRIX FOR A RECTANGULAR
C
       COMMON/BIG/AI,ZETAX,ZETAY,XMAX,YMAX,ETA,XM,IEND,JEND,DYDX,M,NN,
     1CON,A,BBBB,B(20,20),C(20,20),T(20,20),D,Y(20),IP(20),V(20),DX,DY,
     2 AA(20),OUT(20),IOUT,JOUT,KOUT,IT,IDUCT,KEND,DB
      COMPLEX B,C,T,D,Y,V,AI,ZETAX,ZETAY ,C1, A,A1,B1,AA
      REWIND 1
      DO 6 K=1,IEND
      DO 5 I=1,JEND
      C(   I,I)=A+2.*CON*XM*AI*DYDX*DY
      DO 5 J=1,JEND
      IF(J.NE.I) C(   I,J)=0.
      B(   I,J)=0.
      IF(I.EQ.J) B(   I,J)=BBBB
      IF(J.EQ.I+1) B(   I,J)=-1.
      IF(J.EQ.I-1) B(   I,J)=-1.
    5 CONTINUE
C
C  INCLUDE BOUNDARY DATA CORRESPONDING TO Y=0 AND Y=1
C
      A1=   2.*XM*XM*AI*DYDX/(CON*ZETAY*DX)
      B1=-2.*A1-2.*CON*AI*DY/ZETAY
      C1=A1-2.*XM*DYDX/ZETAY
      A1=A1+2.*XM*DYDX/ZETAY
      B(   1,2)=-2.
      B(   JEND,JEND-1)=-2.
      B(1,1)=B(1,1)-B1
      B(JEND,JEND)=B(JEND,JEND)- B1
      C(1,1)=C(1,1)-C1
      C(JEND,JEND)=C(JEND,JEND)- C1
      IF(K .EQ. IEND) GO TO 6
      DO 100 J=1,JEND
100   AA(J)=A
      AA(1)=AA(1)-A1
      AA(JEND)=AA(JEND)-A1
      WRITE(1) AA,B,C
    6 CONTINUE
C
C  INCLUDE BOUNDARY DATA CORRESPONDING TO X=1
C
      DO 7 I=1,JEND
    7 B(I,I)=B(I,I)-C(I,I)*2.*CON*AI*DX/ZETAX
      DO 8 J=1,JEND
    8 AA(J)=A
      AA(1)=AA(1)-A1
      AA(JEND)=AA(JEND)-A1
      WRITE(1) AA,B,C
      DO 10 I=1,JEND
10    Y(I)=-AA(I)
      REWIND 1
      RETURN
      END
```

```
      SUBROUTINE SETUP2
C
C THIS SUBROUTINE SETS-UP THE FINITE DIFFERENCE MATRIX FOR A CYLINDRICAL
C
       COMMON/BIG/AI,ZETAX,ZETAY,XMAX,YMAX,ETA,XM,IEND,JEND,DYDX,M,NN,
     1CON,A,BBBB,B(20,20),C(20,20),T(20,20),D,Y(20),IP(20),V(20),DX,DY,
     2 AA(20),OUT(20),IOUT,JOUT,KOUT,IT,IDUCT,KEND,DB
      COMPLEX B,C,T,D,Y,V,AI,ZETAX,ZETAY ,C1, A,A1,B1,AA
      REWIND 1
      DO 6 K=1,IEND
      DO 5 I=1,JEND
      C(   I,I)=A+2.*CON*XM*AI*DYDX*DY
      DO 5 J=1,JEND
      IF(J.NE.I) C(   I,J)=0.
      B(   I,J)=0.
      IF(I.EQ.J) B(   I,J)=BBBB
      IF(J .EQ. I+1)                     B(I,J)=-1.-1./FLOAT(2*I)
      IF(J .EQ. I-1)                     B(I,J)=-1.+1./FLOAT(2*I)
    5 CONTINUE
C
C  INCLUDE BOUNDARY DATA CORRESPONDING TO Y=0  AND Y=1
C
      A1=  2.*XM*XM*AI*DYDX/(CON*ZETAY*DX)
      B1=-2.*A1-2.*CON*AI*DY/ZETAY
      C1=A1-2.*XM*DYDX/ZETAY
      A1=A1+2.*XM*DYDX/ZETAY
      B(   1,2)=-2.
      B(   JEND,JEND-1)=-2.
      B(JEND,JEND)=B(JEND,JEND)- B1*(1.+1. /FLOAT(2*JEND))
      C(JEND,JEND)=C(JEND,JEND)- C1*(1.+1. /FLOAT(2*JEND))
      IF(K .EQ. IEND) GO TO 6
      DO 100 J=1,JEND
100   AA(J)=A
      AA(JEND)=AA(JEND)-A1*(1.+1./FLOAT(2*JEND))
      WRITE(1) AA,B,C
    6 CONTINUE
C
C  INCLUDE BOUNDARY DATA CORRESPONDING TO X=1
C
      DO 7 I=1,JEND
    7 B(I,I)=B(I,I)-C(I,I)*2.*CON*AI*DX/ZETAX
      DO 8 J=1,JEND
    8 AA(J)=A
      AA(JEND)=AA(JEND)-A1*(1.+1. /FLOAT(2*JEND))
      WRITE(1) AA,B,C
      DO 10 I=1,JEND
10    Y(I)=-AA(I)
      REWIND 1
      RETURN
      END
```

```
      SUBROUTINE FUNCT(N,ZET,F)
       COMMON/BIG/AI,ZETAX,ZETAY,XMAX,YMAX,ETA,XM,IEND,JEND,DYDX,M,NN,
     1CON,A,BBBB,B(20,20),C(20,20),T(20,20),D,Y(20),IP(20),V(20),DX,DY,
     2 AA(20),OUT(20),IOUT,JOUT,KOUT,IT,IDUCT,KEND,DB
       COMPLEX B,C,T,D,Y,V,AI,ZETAY,ZETAX,C1,A,A1,B1,AA
      DIMENSION ZET(N)
      ZETAY=ZET(1)+AI*ZET(2)
      IF(ZET(1).LT.0.) GO TO 40
      A=-(1.-XM*XM)*DYDX**2-CON*XM*AI*DYDX*DY
      IF(IDUCT.LT.0) CALL SETUP
      IF(IDUCT.EQ.0) CALL SETUP1
      IF(IDUCT.GT.0) CALL SETUP2
      REWIND 4
      WRITE(4) (Y(I),I=1,JEND)
      DO 10 I=1,JEND
   10 Y(I)=0.
      IEND1=IEND-1
      DO 15 J=1,IEND1
      WRITE(4) (Y(I),I=1,JEND)
   15 CONTINUE
      WRITE(4) AA
      CALL BLOCK
      CALL OUTPUT
      DB=0.
      CALL SOUND
      GO TO 50
40    DB=1.
50    F=DB
      RETURN
      END
```

```
C**********************************************************************
C       IDENTIFICATION
C         LU3 - LU FACTORIZATION OF A REAL SQUARE MATRIX
C         FORTRAN SUBROUTINE SUBPROGRAM
C         AEROSPACE RESEARCH LABORATORIES
C         WRIGHT-PATTERSON AFB, OHIO 45433
C       PURPOSE
C         LU3 COMPUTES TRIANGULAR MATRICES L AND U AND A PERMUTATION
C         MATRIX P SATISFYING LU = PA GIVEN AN N-SQUARE REAL MATRIX A.
C         LU3 IS INTENDED FOR USE WITH THE ENTRY POINT LEQS3 TO PRODUCE
C         SOLUTIONS OF THE VECTOR EQUATION AX = B.
C       CONTROL
C
C         DIMENSION A(M,M), IP(N)
C         .
C         .
C         .
C         CALL LU3(N,A,M,IP)
C
C         WHERE
C         N  IS AN INTEGER INPUT VARIABLE, THE ORDER OF THE MATRIX A.
C         A  AS A REAL INPUT ARRAY IS THE MATRIX TO BE TRIANGULARIZED.
C         A  AS A REAL OUTPUT ARRAY IS THE UPPER TRIANGULAR FACTOR U
C            IN A(I,J), I .LE. J, AND THE LOWER TRIANGULAR FACTOR L IN
C            A(I,J), I .GT. J.
C         M  IS AN INTEGER INPUT VARIABLE, THE ROW DIMENSION OF A.
C         IP IS AN INTEGER OUTPUT ARRAY, IP(K), K .LT. N, BEING THE IN-
C            DEX OF THE K-TH PIVOT ROW WITH IP(N) BEING ZERO IF A IS
C            SINGULAR AND DET(P) IF A IS NONSINGULAR.
C       OTHER PROGRAMMING INFORMATION
C         LU3 CONTAINS THE FORTRAN STATEMENT
C
C         DATA NUM/200/
C
C         NUM IS INITIAL ARGUMENT SUPPLIED TO THE SYSTEM SUBROUTINE EN-
C              TERED IN CASE LU3 WAS ENTERED WITH A NONPOSITIVE N.
C         DET(A) MAY BE CALCULATED FROM THE FORMULA
C              DET A = FLOAT(IP(N))*A(1,1)* ... *A(N,N)
C         LEQS3 IS CALLED TO COMPUTE SOLUTIONS OF LINEAR SYSTEMS AFTER
C         TRIANGULARIZATION OF A BY LU3.
C       OTHER PROGRAMS REQUIRED
C         THE SYSTEM SUBROUTINE IS CALLED FOR ERROR TRACING AND TERM-
C         INATION.
C       METHOD
C         THE MATRIX A IS REDUCED IN SITU TO TRIANGULAR PRODUCT FORM
C         USING GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING.
C**********************************************************************
```

```
C       IDENTIFICATION
C         LEQS3 - SOLUTION OF A LINEAR SYSTEM GIVEN A TRIANGULAR FACTOR-
C                 IZATION OF THE COEFFICIENT MATRIX
C         FORTRAN SUBROUTINE SUBPROGRAM
C         AEROSPACE RESEARCH LABORATORIES
C         WRIGHT-PATTERSON AFB, OHIO 45433
C       PURPOSE
C         LEQS3 COMPUTES THE SOLUTION X OF THE LINEAR SYSTEM LUX = PAX
C         = PB WHERE L, U, AND P ARE COMPUTED FROM A BY LU3.
C       CONTROL
C
C         DIMENSION A(M,M), IP(N), B(N)
C         .
C         .
C         .
C         CALL LEQS3(N,A,M,IP,B)
C
C         WHERE
C         N  IS AN INTEGER INPUT VARIABLE, THE ORDER OF THE MATRIX A.
C         A  IS A REAL INPUT ARRAY, THE TRIANGULARIZED COEFFICIENT MATRIX
C            COMPUTED BY LU3.
C         M  IS AN INTEGER INPUT VARIABLE, THE ROW DIMENSION OF A.
C         IP IS AN INTEGER INPUT ARRAY, THE INDEXES OF THE PIVOT ROWS
C            COMPUTED BY LU3.
C         B  AS A REAL INPUT ARRAY IS THE COLUMN VECTOR RIGHT-HAND SIDE B
C         B  AS A REAL OUTPUT ARRAY IS THE COLUMN VECTOR SOLUTION X.
C       OTHER PROGRAMMING INFORMATION
C         LEQS3 IS AN ENTRY POINT TO SUBROUTINE LU3.
C       OTHER PROGRAMS REQUIRED
C         NONE
C       METHOD
C         THE TWO TRIANGULAR SYSTEMS LY = PB AND UX = Y ARE SOLVED IN
C         TURN FOR THE SOLUTION X OF LUX = PB.  NO CHECK IS MADE FOR
C         NONSINGULARITY OF THE MATRIX U.  THIS CHECK IS MADE BY INTER-
C         ROGATING IP(N).  AN ATTEMPT TO SOLVE UX = Y WHERE A IS SINGULAR
C         WILL RESULT IN AN INFINITE OR INDEFINITE QUOTIENT.
C       REFERENCES
C         (1) CLEVE B. MOLER, ALGORITHM 423 - LINEAR EQUATION SOLVER,
C             COMM. ACM 15(1972), P.274.
C         (2) PAUL J. NIKOLAI, THE ARL LINEAR ALGEBRA LIBRARY, ARL
C             TECHNICAL REPORT ARL 71-0137(1971).
C***********************************************************************
      SUBROUTINE LU3(N, A, M, IP, B)
C
C  TRIANGULARIZATION OF A REAL SQUARE MATRIX USING GAUSSIAN ELIMINATION
C
      DIMENSION A(M,M), IP(N), B(N)
       COMPLEX A, B, T
      EQUIVALENCE (KP1, KM1)
      DATA NUM /200/
      IP(N) = 1
      NM1 = N - 1
      IF (NM1) 2, 65, 4
 2       CALL SYSTEM(NUM, 12H1ILLEGAL ARG)
```

```
4     DO 60 K = 1, NM1
         KP1 = K + 1
         L = K
         DO 10 I = KP1, N
            IF ((ABS(REAL(A(I,K))) + ABS(AIMAG(A(I,K)))) .GT.
   *            (ABS(REAL(A(L,K))) + ABS(AIMAG(A(L,K))))) L = I
10       CONTINUE
         IP(K) = L
         IP(N) = ISIGN(1, K - L)*IP(N)
         T = A(L,K)
         A(L,K) = A(K,K)
         A(K,K) = T
         IF (ABS(REAL(T)) + ABS(AIMAG(T))) 15, 13, 15
13       IP(N) = 0
         GO TO 60
15          DO 20 I = KP1, N
               A(I,K) = -A(I,K)/T
20          CONTINUE
            DO 40 J = KP1, N
               T =  A(L,J)
               A(L,J) = A(K,J)
               A(K,J) = T
               IF (ABS(REAL(T)) + ABS(AIMAG(T))) 25, 40, 25
25                DO 30 I = KP1, N
                     A(I,J) = A(I,J) + T*A(I,K)
30                CONTINUE
40          CONTINUE
60    CONTINUE
65    IF (A(N,N) .EQ. (0.0,0.0)) IP(N) = 0
      RETURN
      ENTRY LEQS3
C
C  SOLUTION OF A REAL LINEAR SYSTEM TRIANGULARIZED BY SUBROUTINE LU3
C
      NM1 = N - 1
      IF (NM1) 2, 90, 67
67       DO 70 K = 1, NM1
            KP1 = K + 1
            L = IP(K)
            T = B(L)
            B(L) = B(K)
            B(K) = T
            DO 69 I = KP1, N
               B(I) = B(I) + T*A(I,K)
69          CONTINUE
70       CONTINUE
         DO 80 L = 1, NM1
            KM1 = N - L
            K = KM1 + 1
            B(K) = B(K)/A(K,K)
            T = -B(K)
            DO 75 I = 1, KM1
               B(I) = B(I) + T*A(I,K)
75          CONTINUE
80       CONTINUE
90    B(1) = B(1)/A(1,1)
      RETURN
      END
```

REFERENCES

1. Rice, E. J., "Attenuation of Sound in Soft Walled Circular Ducts," AFOSR-UTIAS Symposium on Aerodynamic Noise, University of Toronto Press, 1969.

2. Baumeister, K. J., "Application of Finite Difference Techniques to Noise Propagation in Jet Engine Ducts," NASA TM X-68261; also presented at ASME Winter Annual Meeting, Detroit, Mich., November 1973.

3. Baumeister, K. J., and Rice, E. J., "A Difference Theory for Noise Propagation in an Acoustically Lined Duct with Mean Flow," Progress in Astronautics and Aeronautics-Aeroacoustics: Jet and Combustion Noise; Duct Acoustics, Vol. 37, edited by H. T. Nagamatsu, J. V. O'Keefe, and I. R. Schwartz, AIAA, New York, 1975, pp. 435-453.

4. Rice, E. J., "Propagation of Waves in an Acoustically Lined Duct with a Mean Flow," Basic Aerodynamic Noise Research, NASA SP-207, 1969, pp. 345-355.

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